Physics I

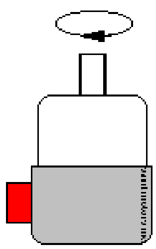
Kinetic Energy

Kinetic energy in linear motion $K.E. = \frac{1}{2}mv^2$

Kinetic energy in rotational motion $K.E. = \frac{1}{2}I\omega^2$

Average kinetic energy in gas molecules due to thermal motion = $\langle KE \rangle = \frac{3}{2}k_B T = \frac{1}{2}mv_{rms}^2$

Problem 1.- Calculate the kinetic energy stored in the rotor of an electric motor whose radius is 0.25m, mass 16kg and angular velocity 3600 rpm. Approximate the moment of inertia of the rotor to one of a solid cylinder: $\frac{1}{2}MR^2$.



Solution: First, we convert $\omega = 3600\frac{\text{rev}}{\text{min}}\left(\frac{1\text{min}}{60\text{s}}\right)\left(\frac{2\pi\text{rad}}{1\text{rev}}\right) = 377 \text{ rad/s}$

Then:

$$K.E._{rotational} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}\left(\frac{1}{2}\times 16\times 0.25^2\right)377^2 = \mathbf{35{,}500\ J}$$

Problem 1a.- Calculate the kinetic energy stored in an audio CD when it rotates at its maximum speed of 500 rpm (revolutions per minute). Approximate the moment of inertia of the CD to one of a disk: $\frac{1}{2}MR^2$, consider its mass to be 0.015kg and its radius 0.060m

Solution: The rotational kinetic energy is given by $K.E. = \frac{1}{2}I\omega^2$, so we need to calculate the moment of inertia and the angular velocity:

Moment of inertia:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.015kg)(0.06m)^2 = 2.7\times 10^{-5}kgm^2$$

If it rotates at 500 rpm, $\omega = 500rpm = 500\frac{rev}{\min}\left(\frac{1\min}{60s}\right)\left(\frac{2\pi}{1rev}\right) = 52.4 \text{ rad/s}$

Then the kinetic energy is: $K.E. = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.7\times 10^{-5}kgm^2)(52.4rad/s)^2 = \mathbf{0.037\ J}$

Problem 1b.- Calculate the total kinetic energy of a barrel that rolls without slipping at 4.0m/s if its mass is 120kg and its radius is 0.35m. Assume that it is a solid cylinder. (Moment of inertia of a solid cylinder =1/2MR^2)

Solution: There are two kinds of kinetic energy, rotational and translational, so we need to add them:

$$K.E. = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2 + \frac{1}{2}Mv^2 = \frac{3}{4}Mv^2$$

Where we replaced the moment of inertia given and $R\omega = v$, so the kinetic energy is:

$$K.E. = \frac{3}{4}Mv^2 = \frac{3}{4}(120kg)(4m/s)^2 = \mathbf{1{,}440\ J}$$

Problem 2.- Calculate the rms speed of a molecule that has a mass of 75 amu if the temperature is 37ºC.
1 amu=1.66×10^{-27} kg

Solution: The kinetic energy is given by $K.E. = \frac{mv^2}{2}$, but also by $< K.E. >= \frac{3}{2}k_BT$, so we can combine the two equations to get the rms speed (average speed):

$$\frac{mv^2}{2} = \frac{3}{2}k_BT \rightarrow v = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times310}{75\times1.66\times10^{-27}}} = \mathbf{320\ m/s}$$

Problem 2a.- At room temperature (300K), a helium atom, with mass 4 amu, typically has a kinetic energy of 6.21×10^{-21} J. Calculate its speed.
1 amu=1.66×10^{-27} kg

Solution: Similar to the previous problem, the kinetic energy is translational, and we get

$$\frac{1}{2}mv^2 = K.E. \rightarrow v = \sqrt{\frac{2K.E.}{m}} = \sqrt{\frac{2(6.21\times10^{-21}J)}{4(1.66\times10^{-27}kg)}} = \mathbf{1{,}370\ m/s}$$

Problem 2b.- In principle, can we separate N_2 from O_2 by diffusion? What is the ratio of speeds of these two molecules at room temperature?

Solution: In principle, yes, we can we separate N_2 from O_2 by diffusion, because the nitrogen molecules are faster by a ratio:

$$\frac{v_{Nitrogen}}{v_{Oxygen}} = \sqrt{\frac{32}{28}} = \mathbf{1.07}$$

Problem 2c.- Calculate the rms speed of hydrogen molecules present in the atmosphere at T=27ºC.

Solution: Since K.E.$=\frac{1}{2}mv_{rms}^2=\frac{3}{2}Nk_BT \rightarrow v_{rms}=\sqrt{\frac{3k_BT}{m}}$

In this equation T = 27+273 = 300K, so:

$$v_{rms}=\sqrt{\frac{3k_BT}{m}}=\sqrt{\frac{3(1.38\times10^{-23}\mathrm{J/K})(300\mathrm{K})}{2\times1.66\times10^{-27}\mathrm{kg}}}= \mathbf{1{,}930\ m/s}$$

Problem 3.- Calculate the kinetic energy stored in a rotating hydrogen molecule that has a moment of inertia of I = 9.56×10^{-48}kgm^2 and is in the first rotational excited state that has an angular momentum squared $L^2 = 2\hbar^2$, where $\hbar = 1.05\times10^{-34}\,Js$

Solution: The rotational kinetic energy is given by: $E=\frac{1}{2}I\omega^2=\frac{L^2}{2I}$, so all we need to do is calculate this energy with the given value of L^2.

$$E=\frac{L^2}{2I}=\frac{2(1.05\times10^{-34}\,Js)^2}{2(9.56\times10^{-48}\,kgm^2)} = \mathbf{1.16\times10^{-21}J}$$

This is a very small quantity but converted to temperature (after dividing by Boltzmann constant) it is 84K!, which is higher than liquid nitrogen.

Problem 4.- Which has more energy: An 80-kg athlete running at 8.5m/s or a 7-g bullet at 350 m/s?

Solution:

$$\mathrm{K.E.}_{athlete}=\frac{1}{2}mv^2=\frac{1}{2}(80\mathrm{kg})(8.5\mathrm{m/s})^2=\mathbf{2{,}890\ J}$$

$$\mathrm{K.E.}_{bullet}=\frac{1}{2}mv^2=\frac{1}{2}(0.007\mathrm{kg})(350\mathrm{m/s})^2=\mathbf{429\ J}$$

The athlete has more kinetic energy!

Problem 5.- A person works out in the stair machine for half an hour. The work done is equivalent to a real climb of 150 meters. Calculate how many Calories were burnt if the person has a mass of 60kg and the efficiency is 20%.

Solution: In a real climb of 150 meters the work would be against gravity and its value can be calculated by multiplying the weight times the height:

$$W = 60\text{kg} \times 9.8\frac{\text{m}}{\text{s}^2} \times 150\text{m} = 88{,}200\text{J}$$

However, the efficiency is 20%, so the energy spent doing that work is

$$E = \frac{W}{\eta} = \frac{88,200}{20\%} = 441{,}000\text{J}$$

We convert this to calories:

$$E = 441{,}000\text{J}\left(\frac{1\text{Cal}}{4{,}184\text{J}}\right) = \mathbf{105\ Cal}$$