Physics I

Potential function

Problem 1.- What would be the kinetic energy of a 0.454kg object that falls straight towards the earth from a height of h = 4×10^6 m when it reaches the surface of our planet?
Ignore air resistance and assume initial velocity zero.
Mass of the Earth M = 5.98×10^{24} kg. Radius of the Earth R = 6.38×10^6 m

Solution: The change in potential energy is converted into kinetic energy, so:

$$K.E. = -G\frac{mM}{r_2} + G\frac{mM}{r_1} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

With the values given:

$$K.E. = 6.67\times10^{-11}\times5.98\times10^{24}\times0.454\left(\frac{1}{6.38\times10^6} - \frac{1}{10.38\times10^6}\right) = \mathbf{1.09\times10^7\ J}$$

Problem 2.- The potential energy for an electron in a quantum dot is described by the equation:

$$U = \frac{a}{r^2} - \frac{b}{r}$$

a) Calculate the point r_o where the potential energy reaches its minimum.
b) Find the binding energy (U at the minimum point).

Solution: The potential energy for an electron in a quantum dot is described by the equation:

$$U = \frac{a}{r^2} - \frac{b}{r}$$

a) The point r_o where the potential energy reaches its minimum

$$\frac{d}{dr}U = -\frac{2a}{r^3} + \frac{b}{r^2} = 0 \rightarrow r_\circ = \frac{2a}{b}$$

b) The binding energy (U at the minimum point)

$$U(r_\circ) = \frac{a}{r^2} - \frac{b}{r} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\frac{2a}{b}} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

Problem 3.- How high will a projectile get if it is launched straight up with an initial velocity of 5,000 m/s from the surface of the Earth. Ignore air resistance and take $M_{Earth} = 5.98\times10^{24}kg$ and $R_{Earth} = 6.38\times10^6 m$

Solution: The projectile will get to a height such that all its initial kinetic energy is traded for potential energy:

$$P.E._1 + K.E._1 = P.E._2 + K.E._2$$

At the turning point $K.E._2 = 0$ and we also notice that $r_1 = R_{Earth}$ and $r_2 = R_{Earth} + h$, so:

$$-G\frac{M_{Earth}m}{R_{Earth}} + \frac{1}{2}mv_1^{\ 2} = -G\frac{M_{Earth}m}{R_{Earth}+h}$$

Dividing by the mass of the projectile:

$$-G\frac{M_{Earth}}{R_{Earth}} + \frac{1}{2}v_1^{\ 2} = -G\frac{M_{Earth}}{R_{Earth}+h}$$

Solving for h:

$$h = \frac{GM_{Earth}}{\frac{GM_{Earth}}{R_{Earth}} - \frac{1}{2}v_1^{\ 2}} - R_{Earth} = R_{Earth}\left[\frac{1}{1-\frac{R_{Earth}}{GM_{Earth}}\frac{1}{2}v_1^{\ 2}} - 1\right]$$

With the values of the problem:

$$h = 6.38\times10^6 m\left[\frac{1}{1-\frac{6.38\times10^6 m}{6.67\times10^{-11}\left(5.98\times10^{24}\right)}\frac{1}{2}(5000m/s)^2} - 1\right] = \mathbf{1.59\times10^6 m}$$