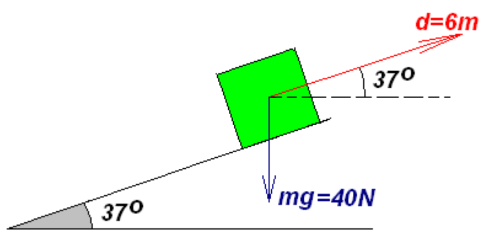
Physics I

Work

Problem 1.- A box whose weight is 40 N is pulled 6.0 m along a 37º inclined plane. What is the work done by the weight of the box?

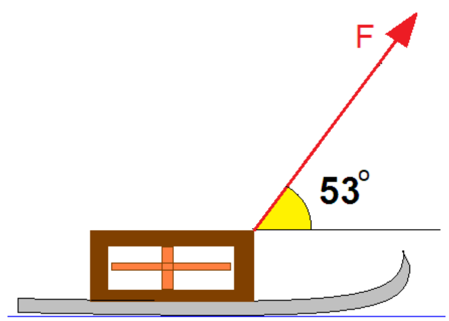
Solution: A figure will help solve this problem:



Notice that the angle between the displacement and the force is 90º + 37º = 127º, so the work done is:

$W = (40N)(6m)\cos 127^{\circ} =$ **-144 J**

Problem 2.- A person pulls a sled on an icy surface with a force of 70.0 N at an angle of 53.0º upward from the horizontal. If the sled moves 12.5 m horizontally, what is the work done by the person?



Solution: $W = Fd\cos\theta = 70N(12.5m)\cos 53^{\circ} =$ **526 J**

Problem 3.- You lift a book up in the air 1.1 m. The mass of the book is 1.2 kg. What is the work done by the weight of the book?

Solution: By definition

$\mathrm{W} = \mathrm{Fd}\cos\theta = 1.2kg(9.8m/s^2)(1.1m)\cos 180^{\circ} =$ **-13 J**

Problem 4.- A 12 kg mass is moving with a speed of 5.0 m/s. How much work is required to stop the mass?

Solution: The work is equal to the change in kinetic energy (final minus initial).

$$\text{W} = -\frac{1}{2}mv^2 = -\frac{1}{2}(12kg)(5m/s)^2 = \mathbf{-150\ J}$$

Problem 5.- A pellet of mass 2 g is shot horizontally into a sandbag, striking the sand with a velocity of 600 m/s. It penetrates 20 cm. What is the average stopping force acting on the pellet?

Solution: The work done by the force will be equal to the change in kinetic energy:

$$\text{W} = -\text{Fd} = -\frac{1}{2}mv^2 \rightarrow F = \frac{mv^2}{2d} = \frac{0.002kg(600m/s)^2}{2(0.2m)} = \mathbf{1{,}800\ N}$$

Problem 6.- What work is required to stretch a spring of constant 40 N/m from x = 0.20 m to 0.25 m? Assume the un-stretched position is at x = 0.

Solution: The work done will be equal to the difference in potential energy:

$$\text{W}=\frac{1}{2}\text{kx}_2{}^2-\frac{1}{2}\text{kx}_1{}^2=\frac{1}{2}40\frac{\text{N}}{\text{m}}(0.25\text{m})^2-\frac{1}{2}40\frac{\text{N}}{\text{m}}(0.2\text{m})^2= \mathbf{0.45\ J}$$

Problem 7.- A skier pushes off the top of a hill with an initial speed of 4.0 m/s. Neglecting friction, how fast will she be moving after dropping 10.0 m in elevation.

Solution: The drop in potential energy will equal the change in kinetic energy:

$$\text{m}gh = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 \rightarrow v_2 = \sqrt{2gh + v_1{}^2} = \sqrt{2(9.8m/s^2)(10m) + (4m/s)^2} = \mathbf{14.5m/s}$$

Problem 8.- A roller coaster starts from rest at a point 45 m above the bottom of a dip. Neglecting friction, what will be its speed at the top of the next slope, which is 30 m above the bottom of the dip?

Solution: The change in height is 15m and the change in potential energy transforms to kinetic energy, so:

$$\text{m}g(15m) = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2g(15m)} = \sqrt{2(9.8m/s^2)(15m)} = \mathbf{17.1m/s}$$

Problem 9.- A pendulum of length 50 cm is pulled 30 cm away from the vertical axis and released from rest. What will be its speed at the bottom of its swing?

Solution: Since the pendulum was pulled 0.3m and the length is 0.5m, the change in height is:

$$h = 0.5m - \sqrt{(0.5m)^2 - (0.3m)^2} = 0.1m$$

The potential energy transforms to kinetic energy, so:

$$\mathrm{m}g(0.1m) = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2g(0.1m)} = \sqrt{2(9.8m/s^2)(0.1m)} = \textbf{1.4 m/s}$$

Problem 10.- The kinetic friction force between a 10-kg object and an horizontal surface is 50.0 N. If the initial speed of the object is 25.0 m/s, what distance will it slide before stopping?

Solution: The work done by the friction force is equal to the change in kinetic energy of the object:

$$\mathrm{W} = -\mathrm{Fd} = -\frac{1}{2}mv^2 \rightarrow d = \frac{mv^2}{2F} = \frac{10kg(25m/s)^2}{2(50N)} = \textbf{62.5 m}$$

Problem 11.- A box is released from rest at the top of a plane inclined 20º above the horizontal. The coefficient of kinetic friction is 0.20. What will be the speed of the mass after sliding 4.0 m along the plane?

Solution: Perpendicular to the inline plane there is no acceleration, so:

$$\mathrm{F_N} = \mathrm{mg}\cos 20^\circ$$

That is, the normal force is equal to the component of the weight in the rotated Y axis.
Let us calculate the acceleration in the direction of motion: There are two forces, the component of the weight in the rotated X axis and the friction force opposing this motion:

$$\mathrm{m}a = mg\sin 20^\circ - F_{friction} = mg\sin 20^\circ - \mu_k F_N = mg\sin 20^\circ - \mu_k mg\cos 20^\circ$$

$$\rightarrow a = g\sin 20^\circ - \mu_k g\cos 20^\circ = 9.8m/s^2(\sin 20^\circ - 0.2\cos 20^\circ) = 1.5\mathrm{m/s^2}$$

Knowing the acceleration and the distance traveled, we can calculate the final speed:

$$v_2^2 = v_1^2 + 2ax \rightarrow v_2 = \sqrt{v_1^2 + 2ax} = \sqrt{2(1.5m/s^2)(4.0m)} = \textbf{3.5m/s}$$

Problem 12.- At what rate is a 60.0 kg girl using energy when she runs up a flight of stairs 10.0 m high in 8.0 s?

Solution: By definition, power is work divided by time:

$$Power = \frac{work}{time} = \frac{mgh}{time} = \frac{60kg(9.8m/s^2)(10.0m)}{8.0s} = \textbf{735W}$$

Problem 13.- A cyclist does work at 600 W while riding. How much force is applied on the bicycle if its speed is 8.0 m/s?

Solution: Notice that power is also equal to force times velocity, so:

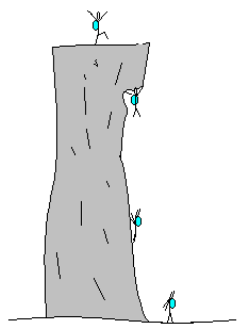
$$600W = F(8m/s) \rightarrow F = \mathbf{75N}$$

Problem 14.- A particle moves from x = 2.0m to x = 6.0m under the influence of a force F=1+x+3x^2. Find the work done by F.

Solution:

$$\mathrm{W} = \int_2^6 1 + x + 3x^2 dx = x + \frac{x^2}{2} + x^3 \Big|_2^6 = \left(6 + \frac{6^2}{2} + 6^3\right) - \left(2 + \frac{2^2}{2} + 2^3\right) = \mathbf{228\ J}$$

Problem 15.- Calculate the amount of work done by a 65kg-rock climber who starts at base camp (altitude 1,100 m) and gets to the summit (altitude 1,250 m). Consider the mass of the gear she is carrying to be 35kg and calculate for 100% efficiency. Give your answer in Calories.
1 Calorie = 4,184 J



Solution: If we had 100% efficiency, all the work she would need to do is to change the potential energy:

$$W = mgh = (35kg + 65kg) \times 9.8 \times 150 = 147{,}000 \text{ J}$$

In calories: $$W = 147{,}000J\left(\frac{1Cal}{4{,}184J}\right) = \mathbf{35\ Cal}$$

Problem 16.- In building the pyramids of Egypt a theory proposes that 20 people would pull a 2,500 kg block up an incline at a 15° angle. Neglecting friction estimate the force applied by each person.

Solution: The total weight is

2,500 × 9.8 = 24,500N

But due to the incline, you only need to pull with a force of 24,500×sin15º = 6341N. Now, if we divide by 20 people each person will pull **320N**.

Problem 17.- An object is moving on a rough, level surface. It has initially 38 J of kinetic energy. The friction force is a constant 2.55 N. How far will it slide?

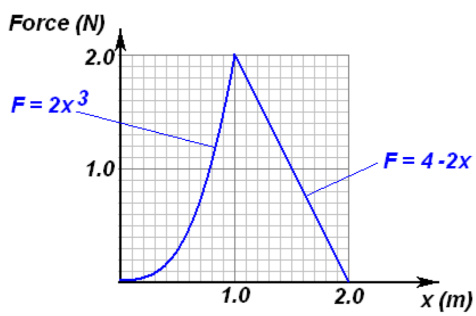
Solution: The energy dissipated through friction is equal to the change in kinetic energy, so:

$$F_{friction} d = \frac{1}{2}mv^2 \rightarrow d = \frac{\frac{1}{2}mv^2}{F_{friction}}$$

The problem specifies the energy and friction force, giving:

$$d = \frac{\frac{1}{2}mv^2}{F_{friction}} = \frac{38J}{2.55N} = \mathbf{14.9\ m}$$

Problem 18.- Calculate the work done by a force described by the following graph when it accelerates an object from x=0 to x=2m:



Solution: We need to integrate the force to find the work:

From $x = 0$ to $x = 1$ the integral is $\int_0^1 2x^3 dx = \left.\frac{x^4}{2}\right|_0^1 = 0.5J$

From $x = 1$ to $x = 2$ the integral is just the area of the triangle 1J
So the work done is **1.5J**

Problem 19.- A weightlifter bench presses 80kg (approx. 175lbs.) 0.75m straight up. How much work does she do, assuming constant velocity, in one lift (just the 0.75m straight up)?

Solution: $W = Fd\cos\theta = 80kg(9.8m/s^2)(0.75m)\cos 0^\circ =$ **588 J**

Problem 20.- An elevator whose total mass is 1,400kg accelerates uniformly from zero to 2m/s upwards in 10 seconds. Calculate the work done in that time.

Solution: The work done on the elevator is equal to change in its mechanical energy. That is the change in potential energy (work done against gravity) and the kinetic energy gained by the elevator.

In equations: $W = mgh + \frac{1}{2}mv^2$

To calculate h we use the average velocity multiplied by the time:

$$h = \frac{v_1 + v_2}{2}t = \frac{0+2}{2}(10) = 10\text{m}$$

Then:

$$W = 1400 \times 9.8 \times 10 + \frac{1}{2}1400 \times 2^2 = \mathbf{140{,}000\ J}$$

Problem 21.- A particle is confined to move following the trajectory $y = x^2$. Calculate the work done on the particle by a force F = (4xy, 9-y) when moving from (1, 1) to (3, 9).

Solution: The work done can be expressed by an integral.

$$W = \int \vec{F}.d\vec{r} = \int F_x dx + F_y dy = \int 4xydx + (9-y)dy$$

It is convenient to express y in terms of x, and integrate:

$$W = \int 4xydx + (9-y)dy = \int_1^3 4x^3dx + (9-x^2)2xdx = \int_1^3 2x^3 + 18xdx$$

$$W = \left.\frac{x^4}{2} + 9x^2\right|_1^3 = \frac{3^4}{2} + 9\times 3^2 - \frac{1^4}{2} - 9\times 1^2 = \mathbf{112\ J}$$