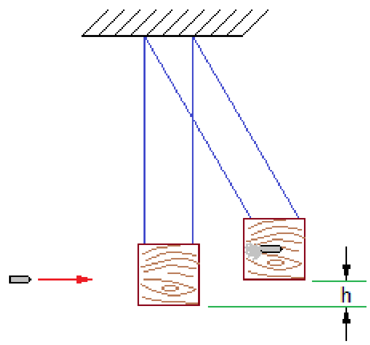
Physics I

Collisions

Conservation of momentum in collisions: $m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$

Conservation of energy in 1-dimensional collisions: $v_1 + v'_1 = v_2 + v'_2$

Problem 1.- A 7-gram bullet with an initial velocity of 250m/s impacts a 693-gram block of wood that was at rest hanging from strings. The bullet and block of wood stay together after the impact and swing upwards. What is the maximum height reached in the swing?



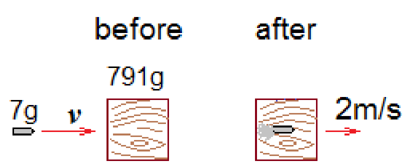
Solution: The velocity after the collision can be calculated using conservation of linear momentum

$$7\times 250 = (7+693)v' \rightarrow v' = \frac{7\times 250}{7+693} = 2.5m/s$$

And conservation of energy after the impact

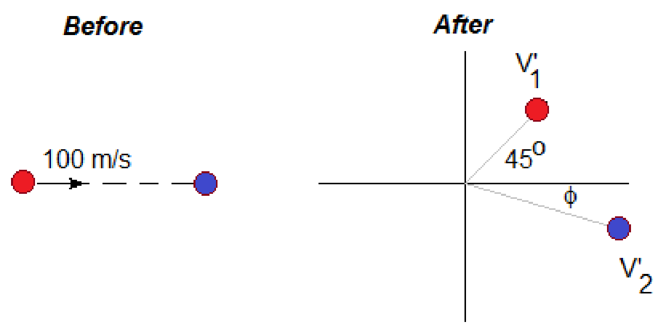
$$\frac{1}{2}mv^2 = mgh \rightarrow h = \frac{v^2}{2g} = \frac{2.5^2}{2\times 9.8} = \quad \mathbf{0.32\ m}$$

Problem 2.- A bullet of mass 7g with velocity $\boldsymbol{v}$ hits a block of wood of mass 791g initially at rest. After the collision the bullet remains embedded in the block and they move together with a final velocity of 2m/s, what was the initial velocity $\boldsymbol{v}$?



Solution: Conservation of momentum $7v = (7+791)\times 2 \rightarrow v =$ **228 m/s**

Problem 3.- A particle of mass 1kg with an initial velocity of 100 m/s collides elastically with a second particle of mass m_2 that was initially at rest.
After the collision the first particle has a velocity that makes an angle of 45 degrees with respect to its initial velocity as shown in the figure.
Calculate the velocity of the two particles after the collision for masses m_2 =2kg, 3kg, … 7kg.



Solution: Conservation of momentum in *x*: $100 = v' + m_2 v'_{2x}$
Conservation of momentum in *y*: $0 = v' + m_2 v'_{2y}$
Conservation of energy: $5000 = \frac{1}{2}(v'^2 + v'^2) + \frac{1}{2} m_2 (v'_{2y}{}^2 + v'_{2x}{}^2)$
Substituting the first two equations in the third we get

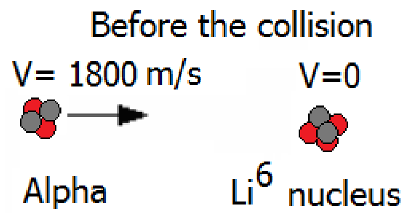
$$5000 = v'^2 + \frac{1}{2} m_2 \left(\frac{v'^2}{m_2{}^2} + \left(\frac{100 - v'}{m_2} \right)^2 \right)$$

This equation can be simplified to give: $(m_2 + 1)v'^2 - 100v' + 5000 - 5000m_2 = 0$

Which can be solved as a quadratic equation as follows: $v' = 50\left(\frac{1 + \sqrt{1 + 2(m_2 + 1)^2}}{(m_2 + 1)} \right)$, where we ignore the negative solution. Knowing the value of v' we can find the other velocities and the angle phi.

m1	1	1	1	1	1	1
m2	2	3	4	5	6	7
v'	89.31	84.31	81.41	79.53	78.21	77.24
v'2x	5.343	5.231	4.646	4.093	3.631	3.252
v'2y	-44.7	-28.1	-20.4	-15.9	-13	-11
phi	-83.2	-79.5	-77.1	-75.6	-74.4	-73.6

Problem 4.- An alpha particle (mass = 4u) with an initial velocity of 1800 m/s collides head on and elastically with a lithium nucleus (mass = 6u) initially at rest. Find the velocity of the lithium nucleus after the collision.



Solution: This is an elastic collision, so linear momentum and kinetic energy are conserved. These are the two equations:

Conservation of momentum

$$p_{before} = p_{after} \rightarrow m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

Conservation of energy

$$v_1 + v'_1 = v_2 + v'_2$$

With the data of the problem the first equation is:

$$4u(1800) + 6u(0) = 4u(v'_1) + 6u(v'_2)$$

Simplifying: $7200 = 4v'_1 + 6v'_2$ Equation 1

The second equation is:

$1800 + v'_1 = 0 + v'_2 \rightarrow 1800 + v'_1 = v'_2$ Equation 2

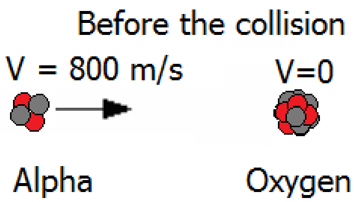
Substituting equation 2 in 1 gives us:

$$7200 = 4(-1800 + v'_2) + 6v'_2$$

$$14{,}400 = 10v'_2 \rightarrow v'_2 = \mathbf{1{,}440 m/s}$$

So, the velocity of the lithium nucleus will be 1,440m/s towards the right.

Problem 4a.- An alpha particle (mass=4u) with an initial velocity of 800 m/s collides head on and **elastically** with an oxygen nucleus (mass=16u) initially at rest. Find the velocity of the oxygen nucleus after the collision.



Solution: The condition of conservation of momentum says that:

$$m_{alpha}\vec{v}_{alpha} + m_O\vec{v}_O = m_{alpha}\vec{v}'_{alpha} + m_O\vec{v}'_O$$

Given the masses of the two nuclei we can re-write the equation as follows:

$$4u\vec{v}_{alpha} + 16u\vec{v}_O = 4u\vec{v}'_{alpha} + 16u\vec{v}'_O$$

Dividing the equation by $4u$ and considering that the collision is head on, so we only need to deal with one dimension: $v_{alpha} + 4v_O = v'_{alpha} + 4v'_O$

The oxygen nucleus was initially at rest, so $v_O = 0$

$$v_{alpha} = v'_{alpha} + 4v'_O \qquad \text{Equation 1}$$

This collision is elastic (energy is conserved) so we have one additional condition:

$$\vec{v}_O - \vec{v}_{alpha} = -(\vec{v}'_O - \vec{v}'_{alpha})$$

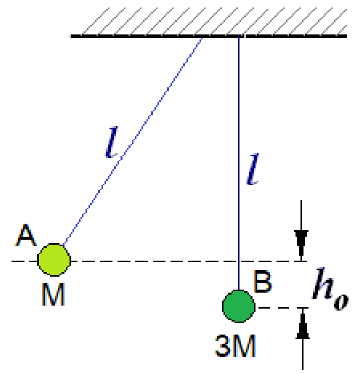
After considering that $v_O = 0$ and the fact that the collision happens in one dimension:

$$-v_{alpha} = -v'_O + v'_{alpha} \qquad \text{Equation 2}$$

We have two equations and two variables; we can subtract Eq.2 from Eq.1 and get:

$$2v_{alpha} = 5v'_O \rightarrow v'_O = \frac{2}{5}v_{alpha} \rightarrow v'_O = \textbf{320 m/s}$$

Problem 5.- Two small spheres of soft, malleable clay, A and B, of mass M and 3M, respectively, hang from the ceiling on strings of equal length l. Sphere A is drawn aside so that it is at a height h_o=1m, as shown in the figure, and then released. It collides with sphere B, they stick together and swing to a maximum height h. Calculate h.



Solution: The plan is to solve this problem in three steps:

i) *Finding the velocity of mass M just before colliding with mass 3M*: We can use conservation of mechanical energy, so: $Mgh_o = \frac{1}{2}\mathrm{M}v^2 \rightarrow v = \sqrt{2gh_o}$

ii) *Finding the velocity of the two masses together just after colliding*: We can use conservation of momentum:

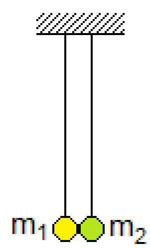
$$Mv = 4Mv' \rightarrow v' = \frac{v}{4} = \frac{\sqrt{2gh_o}}{4}$$

Note: Kinetic energy is not conserved in this collision. It is inelastic.

iii) *Finding the height reached by the two masses together*: We can use conservation of mechanical energy again:

$$(4M)gh = \frac{1}{2}(4M)v'^2 = \frac{1}{2}(4M)\left(\frac{\sqrt{2gh_o}}{4}\right)^2 \rightarrow h = \frac{h_o}{16} = \mathbf{0.0625\ m}$$

Problem 5a.- Two spheres made of soft clay are suspended side by side by 1-m long strings. Their masses are m_1 = 5 g and m_2 = 20 g. If m_1 is pulled 53° off the vertical and released, find their velocities after they collide in a completely inelastic collision.



Problem 6.- An alpha particle (mass = 4*u*) has an elastic collision at $t = 0$ with another nucleus that was initially at rest and bounces straight back with 2/3 of its original speed. Calculate the mass of the second particle.

Solution: The equation of conservation of linear momentum is:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

In the problem $v_A' = -2/3 v_A$ and $v_B = 0$ so:

$$m_A v_A + m_B(0) = m_A(-2/3 v_A) + m_B v_B' \rightarrow m_B = \frac{5/3 m_A v_A}{v_B'}$$

We do not know v_B' but the collision is elastic, so the relative velocity is conserved (with the sign reversed) as shown:

$$v_A - v_B = -[v_A' - v_B']$$

This implies that:

$$v_A - (0) = -[-2/3 v_A - v_B'] \rightarrow v_B' = v_A/3$$

Plugging in this number in the equation above:

$$m_B = \frac{5/3 m_A v_A}{v_B'} = \frac{5/3 m_A v_A}{v_A/3} = 5m_A = \mathbf{20u}$$

Problem 6a.- A 4kg-ball collides elastically with another ball, which is initially at rest. It rebounds in the opposite direction with a speed equal to one third of its original speed. What is the mass of the second ball?

Solution: This is an elastic collision, so linear momentum and kinetic energy are conserved. These are the two equations:

Conservation of momentum

$$p_{before} = p_{after} \rightarrow m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

$$4v_1 + m_2(0)v_2 = 4(-v_1/3) + m_2 v'_2 \rightarrow \frac{16}{3} v_1 = m_2 v'_2$$

Conservation of energy

$$v_1 + v'_1 = v_2 + v'_2 \rightarrow v_1 + (-v_1/3) = 0 + v'_2 \rightarrow v'_2 = 2v_1/3$$

Replacing this in the first equation: $\frac{16}{3}v_1 = m_2(2v_1/3) \rightarrow m_2 =$ **8 kg**

Problem 7.- A proton at rest is hit head on by an alpha particle moving at a speed v. If the collision is elastic, what speed will the proton have after the collision?
Consider the mass of the alpha particle to be 4 times the mass of the proton.

Solution: Similar to the problems above, linear momentum and kinetic energy are conserved. These are the two equations:

Conservation of momentum:
$$4v = 4v'_{alpha} + v'_{proton}$$

Conservation of energy:
$$v = -v'_{alpha} + v'_{proton}$$

Solving for the velocity of the proton:
$$v'_{proton} = \frac{8}{5}v$$

Problem 8.- In a one-dimensional collision, a particle of mass ***m*** with velocity ***v*** collides with a particle of mass 3***m*** at rest. If the particles stick together after the collision, what will be the final velocity of the two particles?

Solution: Momentum is conserved, so:
$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \rightarrow mv + 3m \times 0 = (m+3m)v' \rightarrow v' = \frac{v}{4}$$

Problem 9.- A wooden duck of mass 0.80 kg floating quietly on a pond is hit by a 1g-pellet traveling at 350 m/s that buries itself in the wood. Calculate the speed of the duck just after the hit.

Solution: Since momentum is conserved, we can write:

$$\mathrm{m_{pellet} v_{pellet} + m_{duck} v_{duck} = m_{pellet} v'_{pellet} + m_{duck} v'_{duck}}$$

But the speed of the pellet is equal to the speed of the duck after the hit, so:

$$\mathrm{v' = v'_{pellet} = v'_{duck}}$$

Replacing this in the first equation we get:

$$m_{pellet} v_{pellet} + m_{duck} v_{duck} = (m_{pellet} + m_{duck}) v'$$

With the values given: $v' = \dfrac{(0.001\text{kg})(350\text{m/s})}{(0.001\text{kg} + 0.80kg)} =$ **0.44 m/s**

Problem 10.- A 4g projectile traveling at 450 m/s hits a 1kg block embedding itself in the material. If the block was resting on a smooth surface, find the final velocity of the block and how much kinetic energy was lost.

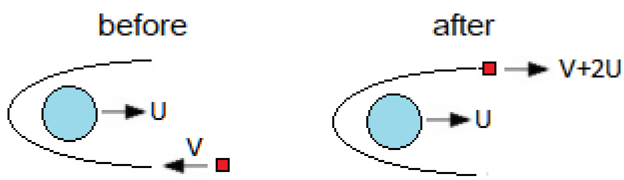
Solution: The final velocity can be calculated from the conservation of linear momentum:

$$v' = \frac{(0.004\text{kg})(450\text{m/s})}{(0.004\text{kg} + 1kg)} = \mathbf{1.793m/s}$$

The energy lost is:

$$\frac{1}{2} \times 0.004 \times 450^2 - \frac{1}{2} \times 1.004 \times 1.793^2 = \mathbf{403.4J}$$

Problem 11.- In the technique called gravitational assistance (slingshot effect), a spacecraft approaches a planet to accelerate. One example is the Cassini probe when it approached the Earth in 1999 on its way to Saturn.
The figure below indicates schematically, how this is accomplished.
Explain why the spacecraft acquires a higher speed and where the extra kinetic energy comes from.



Solution: This can be analyzed as a totally elastic collision.
Conservation of kinetic energy has the consequence that the magnitude of the relative velocity must stay the same before and after.
If the vehicle approaches the planet with a relative velocity -(V+U) it must move away from it with relative velocity V+U. This added to the velocity of the planet gives the spacecraft a final velocity V+2U as shown in the figure above.

The extra kinetic energy of the spacecraft comes from the planet, which loses energy. However, the mass of the Earth is so large compared to the Cassini probe, that nobody has noticed that the years are a bit shorter since 1999.

Problem 12.- When shooting a hunting rifle, the bullet and the rifle must have identical linear momenta just after the shot, however the bullet takes away more kinetic energy. If the mass of the rifle is 1kg and the bullet 5g, what approximate percent of the total kinetic energy is taken by the rifle?

(A) 0.1%
(B) 0.5%
(C) 1%
(D) 5%
(E) 25%

Solution: The kinetic energy can be expressed as $KE = \frac{p^2}{2m}$

Here, p is the linear momentum mv.
Given that p is the same for the bullet and the rifle, the energies will be inversely proportional to their masses. The mass of the rifle is 200 times the mass of the bullet, so it will only take ~0.5% of the total energy.

Problem 13.- If a large truck and a small car collide in an accident, which vehicle experiments the largest force?

Solution: Forces during a collision are opposite in direction, but they have the same magnitude. This is Newton's third law of mechanics.

Problem 13a.- If a large truck and a small car collide in an accident, which vehicle experiments the largest acceleration?

Solution: Forces during the collision have the same magnitude, but the acceleration of the car, which has less mass, is larger.