Physics I

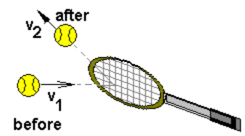
Linear Momentum

Linear momentum (non-relativistic) $\vec{p} = m\vec{v}$

Newton's second law in terms of momentum: $\vec{F} = \dfrac{\Delta\vec{p}}{t}$

Impulse, change in momentum: $\Delta\vec{p} = \int \vec{F}dt$

Problem 1.- In playing a "drop shot" a tennis ball that had an initial velocity of 22m/s horizontally is returned at an angle of 45° above the horizontal with a speed of 7m/s. Calculate the average force on the ball if its mass is 0.057 kg and the contact with the racket lasted 7.5 ms.



Solution: We find the change in momentum and divide by the time to get the force:

Before the ball was hit:

$$\vec{p}_{before} = m\vec{v}_1 = 0.057(22,0)$$

After the ball was hit:

$$\vec{p}_{after} = m\vec{v}_2 = 0.057(-7\cos 45^\circ, 7\sin 45^\circ)$$

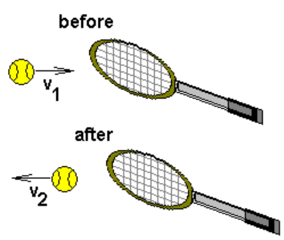
The change in momentum is:

$$\Delta\vec{p} = 0.057(-7\cos 45^\circ, 7\sin 45^\circ) - 0.057(22,0)$$

The force is:

$$\vec{F} = \frac{\Delta\vec{p}}{time} = \frac{0.057(-7\cos 45^\circ, 7\sin 45^\circ) - 0.057(22,0)}{7.5\times 10^{-3}} = (-205, 38)\,\text{N}$$

Problem 1a.- In playing a "drop shot" a tennis ball than had an initial velocity of v_1 = 23m/s horizontally is returned also horizontally with a speed of only v_2 = 2m/s. Calculate the average force on the ball if its mass is 0.057kg and the contact with the racket lasted 7.5ms.



Solution: Like in the previous problem, we find the change in momentum and divide by the time to get the force:

Before the ball was hit:

$$\vec{p}_{before} = m\vec{v}_1 = 0.057(23,\, 0)$$

After the ball was hit:

$$\vec{p}_{after} = m\vec{v}_2 = 0.057(-2,\, 0)$$

The change in momentum is: $\Delta\vec{p} = 0.057(-2,0) - 0.057(24,0)$

The force is:

$$\vec{F} = \frac{\Delta\vec{p}}{time} = \frac{0.057(-2,0) - 0.057(24,0)}{7.5\times10^{-3}} = (-198,0)\text{ N}$$

Problem 2.- A constant 18 N force acts on a 12-kg object for 3.5 s. What is the object's change in velocity?

Solution: Using Newton's second law:

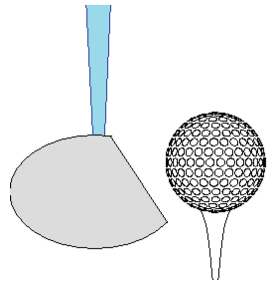
$$F = \frac{\Delta p}{t} = \frac{m\Delta v}{t} \rightarrow \Delta v = \frac{Ft}{m} = \frac{18N(3.5s)}{12kg} = \textbf{5.25 m/s}$$

Problem 2a.- A horizontal force of 230 N is applied to move a 66-kg cart, initially at rest, across a 13 m level surface. What is the final speed of the cart?
Ignore friction in this problem.

Solution: If we ignore friction, all the work done on the cart is converted into kinetic energy, giving:

$$W = Fd\cos\theta = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2Fd\cos\theta}{m}} = \sqrt{\frac{2(230\text{N})(13\text{m})\cos 0^\circ}{66\text{kg}}} = \textbf{9.5m/s}$$

Problem 3.- Calculate the average force that a club imparts on a golf ball if it is hit off the tee with a speed of 45 m/s and the time they are in contact is 2.5ms. Take the mass of the golf ball as 0.046 kg.



Solution: Using Newton's second law:

$$F = \frac{\Delta p}{t} = \frac{0.046kg(45m/s)}{2.5\times10^{-3}s} = \mathbf{828\ N}$$

Problem 3a.- A golf ball of mass 45.9g is hit by a club in a collision that lasts 1.5ms. Estimate the force applied on the ball if it traveled 275m horizontally and its initial velocity was at 45º above the horizontal. Ignore air resistance.

Solution: First, we find the initial velocity of the golf ball (after being hit by the club) using the range equation:

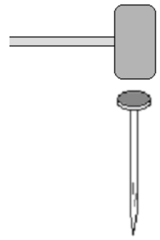
$$R = \frac{v_o^2 \sin(2\theta)}{g} \rightarrow v_o = \sqrt{\frac{Rg}{\sin(2\theta)}}$$

For a range of 275 m: $v_o = \sqrt{\frac{275m(9.8m/s^2)}{\sin(2\times45^\circ)}} = 51.9$ m/s

Knowing the velocity after being hit by the club allows us to find the change in momentum: $\Delta p = mv_o$ and estimate the force using Newton's second law: $F = \frac{\Delta p}{t} = \frac{mv_o}{t}$.

$$F = \frac{0.0458kg(51.9m/s)}{1.5\times10^{-3}s} = \mathbf{1,580\ N}$$

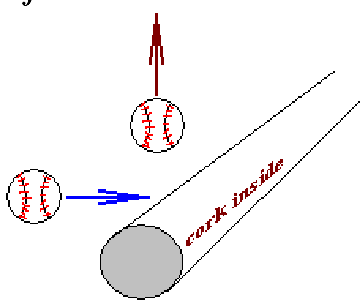
Problem 4.- A 1.5kg hammer with an initial velocity of 1.2m/s hits a nail and slows down to rest in 0.012 s. Calculate the average force that the hammer applies on the nail.



Solution: Using Newton's original second law:

$$F = \frac{\Delta p}{t} = \frac{1.5kg(1.2m/s)}{0.012s} = \mathbf{150\ N}$$

Problem 5.- A baseball that had an initial horizontal velocity of 30m/s is hit straight up with a velocity of 16m/s. Calculate the average force exerted by the bat on the ball if the mass of the ball is 0.140kg and the time of contact was 5ms.
Answer with the magnitude of the force.



Solution: The plan is to calculate the change in momentum $\Delta\vec{p}$ and divide it by the time to get the force.

After the hit the momentum is $\vec{p}_{after} = (0, mv_{after}) = (0, 0.140\times16) = (0, 2.24)$

Before the hit the momentum is $\vec{p}_{before} = (mv_{before}, 0) = (0.140\times30, 0) = (4.2, 0)$

To find the difference: $\Delta\vec{p} = \vec{p}_{after} - \vec{p}_{before} = (-4.2,\ 2.24)$

And to find the force: $\vec{F} = \frac{\Delta p}{t} = \frac{(-4.2,\ 2.24)}{5\times10^{-3}} =$ (-840 N, 448 N)

The magnitude is: $F = \sqrt{840^2 + 448^2} = \mathbf{952\ N}$