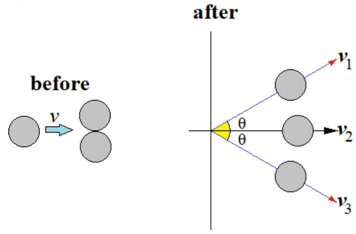
Physics I

Scattering

Problem 1.- A ball with initial velocity v collides elastically and symmetrically with two other identical balls, which are initially at rest. Calculate the velocity of the three balls after the collision if there is no friction during the collision.



Solution: Due to symmetry $v_1 = v_3$ (in magnitude). In addition, during the collision the centers of the three balls form the corners of an equilateral triangle, and the forces between them will be along the lines that connect the centers (the sides of that triangle) because there is no friction, so θ=30°.

Conservation of momentum in the X-direction gives us the equation:

$$mv = mv_2 + 2mv_1 \cos 30^\circ \rightarrow v = v_2 + \sqrt{3}v_1$$

And conservation of kinetic energy in the elastic collision:

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_2{}^2 + 2\frac{1}{2}mv_1{}^2 \rightarrow v^2 = v_2{}^2 + 2v_1{}^2$$

We can solve for v_2 in the first equation and replace it in the second equation

$$v_2 = v - \sqrt{3}v_1$$

$$v^2 = \left(v - \sqrt{3}v_1\right)^2 + 2v_1{}^2$$

Solving for v_1 we get

$$v^2 = v^2 - 2\sqrt{3}vv_1 + 3v_1{}^2 + 2v_1{}^2$$

$$0 = -2\sqrt{3}vv_1 + 5v_1{}^2$$

There are two solutions, the trivial $v_1 = 0$ and

$$v_1 = \frac{2\sqrt{3}}{5}v$$

And $v_2 = v - \sqrt{3}\frac{2\sqrt{3}}{5}v = -\frac{1}{5}v$

Problem 2.- An alpha particle (mass = 4 amu) initially moving at 3,000 m/s horizontally to the right is scattered at 90 degrees off a carbon nucleus (mass = 12 amu) initially at rest. Calculate the velocities of the two particles after the collision.

Solution:

Velocity of alpha particle before the collision: $v = (3000, 0)$
Velocity of alpha particle after the collision: $v = (0, a)$
Velocity of carbon nucleus before the collision: $v = (0, 0)$
Velocity of carbon nucleus after the collision: $v = (b, -c)$

Conservation of momentum in the x-direction: $4\times 3000 = 12b \rightarrow b = 1{,}000$

Conservation of momentum in the y-direction: $0 = 4a - 12c \rightarrow a = 3c$

Conservation of energy $\frac{1}{2}4\times 3000^2 = \frac{1}{2}4\times a^2 + \frac{1}{2}12\times b^2 + \frac{1}{2}12\times c^2$
This last equation can be simplified: $3000^2 = a^2 + 3b^2 + 3c^2$

And substituting the other two equations:

$$3000^2 = (3c)^2 + 3\times 1000^2 + 3c^2 \rightarrow \frac{1000^2}{2} = c^2 \rightarrow c = 707$$

and $a = 3c = 2{,}121$

So, the velocities after the collision are:

$$v'_{alpha} = (0,\ \ 2121)$$

and

$$v'_{carbon} = (1000, -707)$$