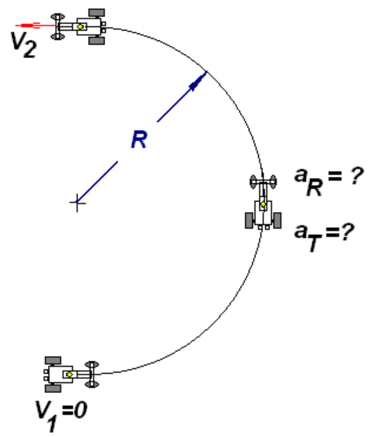
Physics I

Centripetal Acceleration

Problem 1.- A Formula 1 car accelerates uniformly from rest to a speed of v_2=200 km/h by following a semicircle of radius R=350m. Calculate its centripetal and tangential accelerations in the middle of the curve.



Solution: Imagine that you stretch the arc into a straight line, and then the linear acceleration (the *tangential* acceleration) can be calculated using:

$$v_2^2 = v_1^2 + 2ax$$

Where: $x = \pi R = 3.1416 \times 350\text{m} = 1{,}100m$

$v_1 = 0$ and

$$v_2 = 200\frac{km}{h}\left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1km}\right) = 55.6m/s$$

So the tangential acceleration is:

$$a_T = \frac{v_2^2 - v_1^2}{2x} = \frac{(55.6m/s)^2}{2 \times 1100m} = \mathbf{1.4\ m/s^2}$$

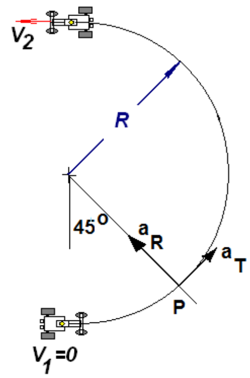
To find the centripetal acceleration we use the equation $a_R = \frac{v^2}{R}$. We already have the radius of the curve, but we need the speed when the car is halfway through the arc. To do this we use the formula $v_2^2 = v_1^2 + 2ax$ again, but this time we only use half the distance in the equation (550m, not 1100m) and solve for v_2.

$$v^2 = v_1^2 + 2ax \rightarrow v = \sqrt{2(1.4m/s^2)(550m)} = 39.2\text{m/s}$$

using this to get the centripetal acceleration we get:

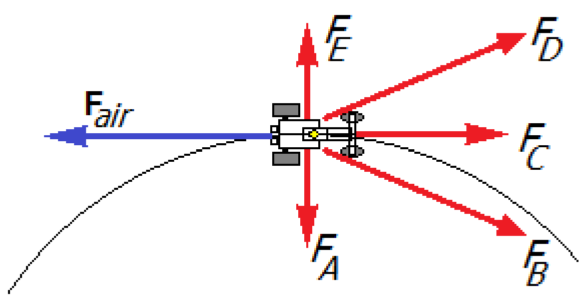
$$a_R = \frac{(39.2m/s)^2}{350m} = \mathbf{4.4\ m/s^2}$$

Problem 1b.- A Formula 1 car accelerates uniformly from rest to a speed of v_2=180 km/h by following a semicircle of radius R=280m. Calculate its centripetal and tangential accelerations when the car reaches point P, which is 45° from the initial point.

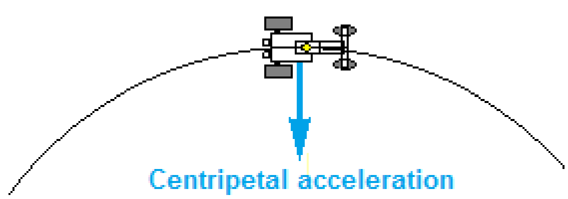


Problem 2.- A car travels with constant speed on a circular road on level ground. In the diagram below, $\mathbf{F_{air}}$ is the force of air resistance on the car. Which of the other forces shown best represents the horizontal force of the road on the car's tires? Give a short rationale of your answer.

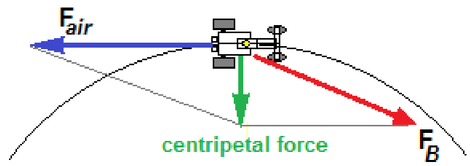
(A) $\mathbf{F_A}$ **(B) $\mathbf{F_B}$** **(C) $\mathbf{F_C}$** **(D) $\mathbf{F_D}$** **(E) $\mathbf{F_E}$**



Solution: Since the speed of the car is constant, there is no tangential acceleration, so the only acceleration is centripetal:



According to the problem there are only two forces acting on the car in the plane of its motion: air resistance and friction force, but the sum of these two forces must be $F=ma$ due to Newton's second law, so the vector diagram will be:



So, the correct answer is (B) **F_B**

Problem 3.- Find the centripetal acceleration (radial acceleration) of a person standing on the Earth equator. Consider the radius of the Earth to be 6.4×10^6 m and the period of rotation to be 1 day [8.6×10^4 s].

Solution: $a_R = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R = \frac{4\pi^2 R}{T^2} = \mathbf{0.043\ m/s^2}$

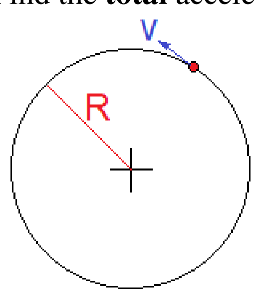
Problem 3a.- Find the period that would make the centripetal acceleration (radial acceleration) of a person standing on the Earth equator equal to "g". Consider the radius of the Earth to be 6.4×10^6 m.

Notice that if the day had this period instead of 24 hours, we would feel weightless at the equator.

Solution: We want: $g = \omega^2 R$, which can be written as: $g = \left(\frac{2\pi}{T}\right)^2 R \rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

Given the values of the problem: $T = 2\pi\sqrt{\frac{6.4\times10^6\,\text{m}}{9.8\,\text{m/s}^2}}$ = **5,080 s**

Problem 4.- The speed of a particle moving in a circle of radius R=8m is given by $v=5t^2+2t$. Where v is in m/s and t is in seconds. Find the **total** acceleration of the particle at t=1s



Solution: The acceleration in the tangential direction is:

$$a_T = \frac{dv}{dt} = 10t + 2 = 12$$

The centripetal acceleration is:

$$a_R = \frac{v^2}{R} = \frac{(5t^2+2t)^2}{R} = \frac{(7)^2}{8} = 6.125$$

And the total acceleration is: $a_{TOTAL} = \sqrt{12^2 + 6.125^2} = \mathbf{13.5\ m/s^2}$

Problem 4a.- A particle is constrained to move in a circle with a 12-meter radius. At one instant, the particle's speed is 6 meters per second and is increasing at a rate of 4 meters per second squared. What is the magnitude of the total acceleration at that instant?

Solution: There are two components of the acceleration:

The centripetal acceleration is $a_R = \frac{\text{v}^2}{\text{R}} = \frac{6^2}{12} = 3\text{m/s}^2$

The tangential acceleration is given directly in the problem: $a_T = 4\text{m/s}^2$

These two components must be added as vectors:

$$a_{Total} = \sqrt{a_T^{\ 2} + a_R^{\ 2}} = \sqrt{3^2 + 4^2} = \mathbf{5\ m/s^2}$$

Problem 5.- Find the centripetal acceleration (radial acceleration) of a pilot pulling out of dive at 550m/s by following a circular trajectory of 8km radius.

Solution: The problem gives the speed v=550m/s and the radius R=8,000 m. Then, the acceleration is:

$$a = \frac{v^2}{R} = \frac{550^2}{8{,}000} = \mathbf{37.8\ m/s^2} \quad \text{… almost 4 “g”s!}$$

Problem 5a.- Find the minimum radius of a circular trajectory of a pilot pulling out of dive at 450m/s if the centripetal acceleration should not exceed 3.5 "g"s.

Solution: The problem gives the speed v=450m/s and the acceleration $a = 3.5 \times 9.8 = 34.3 m/s^2$. Then the radius can be calculated as follows:

$$a = \frac{v^2}{R} \rightarrow R = \frac{v^2}{a} = \frac{450^2}{34.3} = \mathbf{5{,}900\ m}$$

Problem 6.- A clinical centrifuge reaches 7035 rpm in 28 seconds. Calculate the angular acceleration in rad/s^2 assuming it is constant. With this value calculate the *tangential acceleration* of a blood sample located at R=0.12m.

Solution: According to the problem a final angular velocity of is given and reached in 28s, so the angular acceleration can be calculated as $\alpha = \frac{\omega_{final}}{t}$ and then the tangential acceleration will be:

$$a_T = R\alpha = \frac{R\omega_{final}}{t}.$$

Recall that ω_{final} should be in rad/s, so:

$$\omega_{final} = 7035 rpm \left(\frac{1\min}{60s}\right)\left(\frac{2\pi radians}{1rev}\right) = 737 rad/s$$

If the time to reach that velocity is 28s:

$$a_T = \frac{0.12m(737 rad/s)}{28s} = \mathbf{3.14 m/s^2}$$

Problem 6a.- In the problem above, find the centripetal acceleration of the blood sample when the centrifuge reaches 7035 rpm.

Solution: In the problem above the centripetal acceleration is:

$$a_R = \omega^2 R = 737^2 \times 0.12 = \mathbf{65{,}200\ m/s^2}$$

Problem 7.- A modern centaur (a biker and his motorcycle) has a mass of 230 kg. He goes around a 35 m radius turn at 95 km/h. Find the centripetal force.

Solution: The speed of the creature is:

$$v = 95\frac{km}{h}\left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1mile}\right) = 26.4m/s$$

The centripetal acceleration is: $a_R = \frac{v^2}{R} = \frac{(26.4m/s)^2}{35m} = 19.9m/s^2$

The force is equal to *ma*, so if the mass is 230 kg:

$$F = ma_R = (230kg)(19.9m/s^2) = \mathbf{4{,}580\ N}$$

Problem 8.- The radius of a curve in the highway is 220m. What is the maximum possible speed of a vehicle rounding the curve if the centripetal acceleration should not exceed 3.5 m/s^2? Give your answer in miles per hour. [1 mile=1609m].

Solution: The maximum velocity will happen when the car has maximum acceleration:

$$\frac{v^2}{R} = a_R \rightarrow v = \sqrt{Ra_R}$$

With the values of the problem, we get: $v = \sqrt{(220m)(3.5m/s^2)} = \mathbf{27.7m/s}$

Converting to miles per hour: $v = 27.7\text{m/s}\left(\frac{3600\text{s}}{1\text{h}}\right)\left(\frac{1\text{mile}}{1609\text{m}}\right) = \mathbf{62.1\ mph}$

Problem 8a.- What should be the radius of a curve in a highway if the centripetal acceleration should not exceed 3.5 m/s^2 for a car driving at 65 miles/hour? [1 mile=1609m]

Solution: The minimum radius will happen when the car has maximum acceleration:

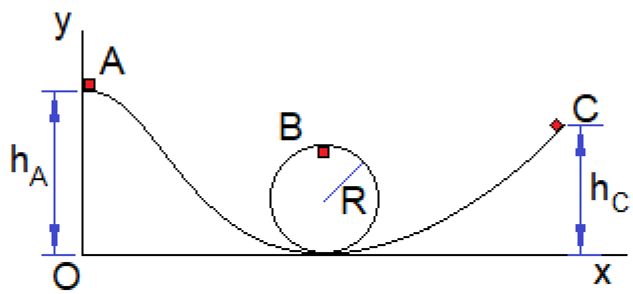
$$\frac{v^2}{R} = a_R \rightarrow R = \frac{v^2}{a_R}$$

Let's convert the speed to m/s:

$$v = 65\frac{\text{mile}}{\text{h}}\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1609\text{m}}{1\text{mile}}\right) = 29.1\text{m/s}$$

With this value we get: $R = \frac{(29.1\text{m/s})^2}{3.5\text{m/s}^2} = \mathbf{242\ m}$

Problem 9.- An object slides without friction on the path shown below.



Its speed is given by the equation:

$$v = \sqrt{2g(h_A - y)} \qquad h_A = 3R$$

a) The magnitude of its acceleration is $a = \frac{g}{2}\sqrt{39}$ when its vertical position is 3/2R when going up inside the loop. Determine the tangential and radial acceleration vectors at that point.

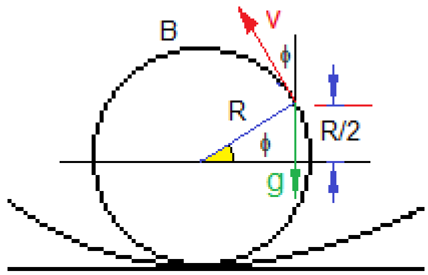
b) Find the acceleration at the vertical position 3R/2, inside the loop, but going down.

c) Indicate at what position A, B or C the radial acceleration is maximum and minimum.

d) Indicate and justify at what position, B or C, the tangential acceleration is largest.

Solution:

a) By geometric considerations we can find the direction of the velocity vector, which has to be tangent to the circular trajectory in the loop at the instant indicated. A graph will help us in this case:



We notice that the sine of angle ϕ is 0.5, so it is 30 degrees.
To find the tangential acceleration we calculate the component of g in the direction of the velocity:

$a_{tangential}$ = g cosϕ = **8.66 m/s^2**

And the vector is:

$\mathbf{a}_{tangential}$ = (8.66 sin ϕ, -8.66 cos ϕ) m/s^2

$\mathbf{a}_{tangential}$ = **(4.33, -7.5) m/s^2**

Using the given equation, we find the speed at the indicated point:

$$v = \sqrt{2g(3R - 3R/2)} = \sqrt{3gR}$$

And the radial acceleration is

a_{radial} = v^2/R=3g = **30 m/s^2**

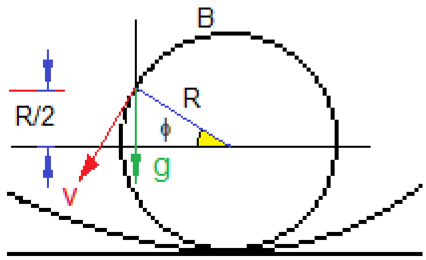
In vector form:

$\mathbf{a}_{radial}$ = (-30 cos ϕ, -30 sin ϕ) m/s^2

$\mathbf{a}_{radial}$ = **(- 26, -15) m/s^2**

Notice that in this method we do not use the given total acceleration, but that piece of information is consistent with our calculations.

b) When descending in the loop, by symmetry the radial and tangential accelerations will be the same in magnitude, but with mirror-image directions. To clarify we prepare another graph:



With the necessary changes we get

$\mathbf{a}_{tangential}$ = **(- 4.33, -7.5) m/s^2** $\mathbf{a}_{radial}$ = **(26, -15) m/s^2**

a = **(21.67, -22.5) m/s^2**

c) Recall that the radial acceleration can be calculated by taking the speed squared divided by the trajectory's curvature radius.

- At point A the speed is zero, so the radial acceleration is also zero.

- At point B the speed is $\sqrt{2gR}$ and the radius is R, so the radial acceleration is 2g.

- At point C the speed is slower than at B because C is higher than B. In addition, this part of the trajectory has a very large radius of curvature (it might even be a straight line at that point). We can safely conclude that the radial acceleration at C is smaller than at B.

Maximum acceleration is at B.
Minimum acceleration is at A.

C would tie A if we could confirm that the trajectory at C is straight.

d) When considering the tangential acceleration, we need to remember that this occurs due to the change in speed.

Then, if you examine the speed equation given you will notice that to have change in speed you need to change the height.
At point B the trajectory is horizontal, so there is no change in height and the tangential acceleration is zero.

At point C the object is climbing, so its height is increasing, its speed is decreasing and the tangential acceleration is in the opposite direction of its velocity.

In conclusion:

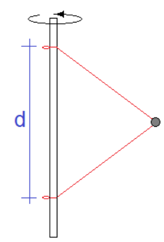
Minimum tangential acceleration (zero) at B.
Maximum tangential acceleration at C.

Another way to look at this problem is to notice that because there is no friction, only the component of the weight in the direction of the velocity produces tangential acceleration.

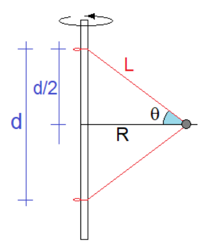
Problem 10.- Are the people inside a space station in orbit around the Earth in equilibrium?

Solution: They are accelerated towards the center of the Earth, so they are not in equilibrium.

Problem 11.- A 2.45kg ball is attached to a rotating pole by two identical massless strings, each of length 1.50m. The strings are tied to the pole separated by a distance d = 1.80m. The tension in the lower string is 10.0N. Calculate:
(a) The tension in the upper string.
(b) The net force on the ball.
(c) The speed of the ball.

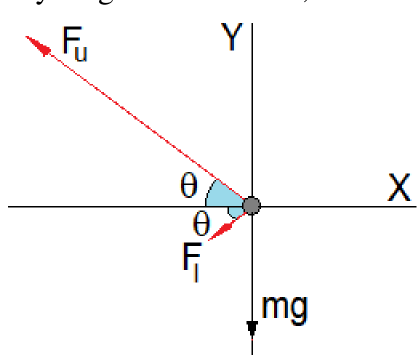


Solution.- First we use trigonometry to find the radius of rotation R and the angle θ in the problem:



$$R = \sqrt{L^2 - (d/2)^2} = \sqrt{1.5^2 - 0.9^2} = 1.2\text{m}$$
$$\theta = \sin^{-1}\left(\frac{d/2}{L}\right) = \sin^{-1}\left(\frac{1.8/2}{1.5}\right) = 37^\circ$$

Next, we can examine the free body diagram of the ball, on which three forces are acting:



In the Y-direction the acceleration is zero, so the sum of the forces must be zero. That allows us to calculate the force on the upper string.

$$\sum F_y = 0$$

$$F_u \sin\theta - F_l \sin\theta - mg = 0$$

$$\rightarrow F_u = \frac{mg + F_l \sin\theta}{\sin\theta} = \frac{2.45\times 9.8 + 10\times \sin 37^\circ}{\sin 37^\circ} = \mathbf{50N}$$

Next, we can add the forces in the X-direction to get the net force on the ball:

$$\sum F_x = -F_u \cos\theta - F_l \cos\theta = -50\times 0.8 - 10\times 0.8 = \mathbf{-48N}$$

This force has to be equal to the centripetal force, so

$$48 = m\frac{v^2}{R} \rightarrow v = \sqrt{\frac{48R}{m}} = \sqrt{\frac{48\times 1.2}{2.45}} = \mathbf{4.8m/s}$$