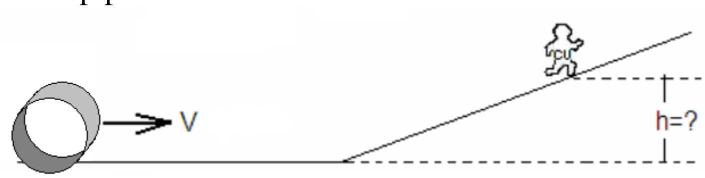
Physics I

Rolling

Problem 1.- An accidentally loosen concrete pipe in a construction site rolls toward you at 8.3m/s on a level surface. To protect your life, you run up an incline. How far up do you need to go to be safe?
The moment of inertia of a pipe is MR^2



Solution: The cylinder will roll until all its kinetic energy is converted to potential energy. If you go higher than that you will be safe. Recall that kinetic energy is the sum of translational plus rotational energies and so the equation will be:

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$

The value of the moment of inertia is MR^2 so:

$$\frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\omega^2 = Mgh$$

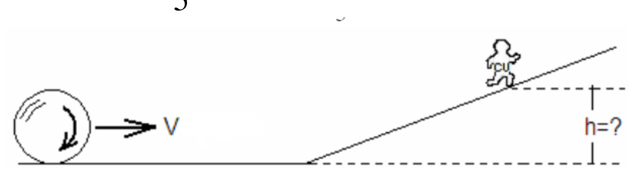
We can divide both sides of the equation by M, and we also notice that $R^2\omega^2 = v^2$ so the equation can be simplified to give:

$$\frac{1}{2}v^2 + \frac{1}{2}v^2 = gh \rightarrow h = \frac{v^2}{g} = \frac{v^2}{g}$$

If v = 8.3 m/s, we get: $h = \frac{(8.3m/s)^2}{9.8m/s^2} =$ **7.0 m**

Problem 1a.- A giant spherical ball rolls toward you at 5m/s on a level surface. In order to protect your life, you run up an incline. How far up do you need to go to be safe?
Suggestion: use conservation of energy. The snowball will stop when all its kinetic energy, rotational and linear, is converted to potential energy.

The moment of inertia of a sphere is $\frac{2}{5}MR^2$



Solution: Like the previous problem, the ball will roll until all its kinetic energy is converted to potential energy. If you go higher than that you will be safe. Recall that kinetic energy is the sum of translational plus rotational energies and so the equation will be:

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$

The value of the moment of inertia is $\frac{2}{5}MR^2$ so:

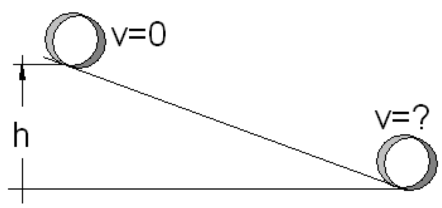
$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = Mgh$$

We can divide both sides of the equation by M, and we also notice that $R^2\omega^2 = v^2$ so the equation can be simplified to give:

$$\frac{1}{2}v^2 + \frac{1}{2}\left(\frac{2}{5}\right)v^2 = gh \rightarrow h = \frac{\frac{1}{2}v^2 + \frac{1}{2}\left(\frac{2}{5}\right)v^2}{g} = \frac{(7/10)v^2}{g}$$

With the given value of v, we get: $h = \frac{(7/10)(5m/s)^2}{9.8m/s^2} =$ **1.8 m**

Problem 2.- A hoop rolls down an incline without slipping. Find its linear velocity after falling h=9.8m vertically. Assume its initial velocity was zero.
The moment of inertia of a hoop about its center is MR^2.



Solution: Since the friction force and the normal force don't do any work, mechanical energy is conserved, which means that the potential energy lost is converted into kinetic energy. Remember that you have two kinds of kinetic energy: linear and rotational.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Let's replace in the equation the value of the moment of inertia:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\omega^2$$

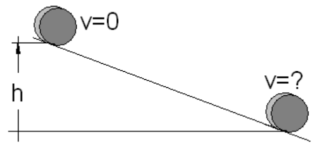
But $v = \omega R$ so:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

Solving for v:

$$v = \sqrt{gh} = \sqrt{9.8 \times 9.8} = \textbf{9.8 m/s}$$

Problem 2a.- A solid cylinder rolls down an incline without slipping. Find its linear velocity after falling h=9.8m vertically. Assume its initial velocity was zero.
The moment of inertia of a cylinder about its center is $1/2MR^2$.



Solution: Like the previous problem, since friction and the normal force don't do any work, mechanical energy is conserved, which means that the potential energy lost is converted into kinetic energy. You have two kinds of kinetic energy: linear and rotational.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

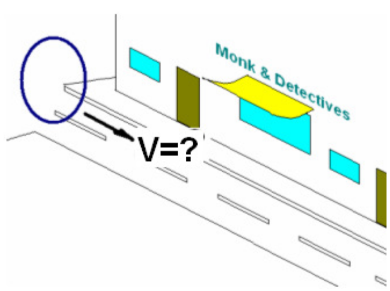
Let's replace in the equation the value of the moment of inertia:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2$$

But $v = \omega R$ so: $$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

Solving for v: $$v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4}{3}9.8 \times 9.8} = \textbf{11.3 m/s}$$

Problem 2b.- Lombard Street in San Francisco, California has a section that has a steep slope. Calculate the velocity of a **hoop** rolling down the incline after falling 5 m vertically. Assume the hoop started from rest.
Moment of inertia of a hoop is MR^2.



Solution: Conservation of energy:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

$$\rightarrow gh = v^2$$

The speed after falling 5m is: $v = \sqrt{gh} = \sqrt{(9.8m/s^2)(5m)} =$ **7.0 m/s**

Problem 3.- If a sphere and a solid cylinder roll down a slope, which one accelerates faster?

Solution: You can prove that the equation for the acceleration rolling going down a slope with angle θ is

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$$

So, the sphere will accelerate faster than the solid cylinder, because it has less moment of inertia compared to its mass times its radius squared.

$$a_{sphere} = \frac{g\sin\theta}{1 + \frac{\frac{2}{5}mR^2}{mR^2}} = \frac{g\sin\theta}{1.4}$$

$$a_{solid\ cylinder} = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}mR^2}{mR^2}} = \frac{g\sin\theta}{1.5}$$

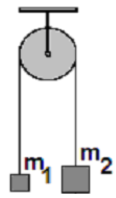
Problem 3a.- What about a hollow cylinder and a solid one? Which one will accelerate faster when rolling down a slope?

Solution: Like the problem above, we compare the accelerations.

$$a_{hollow\ cylinder} = \frac{g\sin\theta}{1 + \frac{mR^2}{mR^2}} = \frac{g\sin\theta}{2} \qquad a_{solid\ cylinder} = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}mR^2}{mR^2}} = \frac{g\sin\theta}{1.5}$$

The solid cylinder will be faster.

Problem 4.- An Atwood machine is made with two masses M_1=10kg and M_2=11kg. The pulley has a mass of 2kg, and it has a shape approximately the same as a disk.
Calculate how long it will take mass M_2 to fall 1m starting from rest.
The moment of inertia of a disk about its center is $1/2MR^2$.



Solution: The acceleration can be determined by writing the equation of conservation of energy:

$$(m_2 - m_1)gh = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2$$

Now we take derivative with respect to time:

$$(m_2 - m_1)gv = \frac{1}{2}m_1 2va + \frac{1}{2}m_2 2va + \frac{1}{2}I2\omega\alpha$$

We multiply by R^2/R^2 in the last term and cancel v, so:

$$(m_2 - m_1)g = m_1 a + m_2 a + \frac{Ia}{R^2}$$

The acceleration is $a = \dfrac{m_2 - m_1}{m_2 + m_1 + \dfrac{I}{R^2}} g = \dfrac{11-10}{11+10+1} \times 9.8 = 0.445\text{m/s}^2$

The time: $t = \sqrt{\dfrac{2x}{a}} = \sqrt{\dfrac{2}{0.445}} =$ **2.1s**