Physics I

Rotational Kinematics

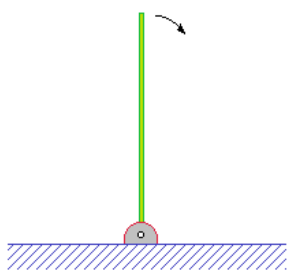
Equations for constant angular acceleration:

$$\langle \omega \rangle = \frac{\theta}{t} = \frac{\omega_1 + \omega_2}{2} \qquad \text{average angular velocity}$$

$$\omega_2 = \omega_1 + \alpha t \qquad \theta = \omega_1 t + \frac{1}{2}\alpha t^2 \qquad \omega_2^{\ 2} = \omega_1^{\ 2} + 2\alpha\theta$$

Rotational kinetic energy $\quad KE_{\text{rotational}} = \frac{1}{2} I\omega^2$

Problem 1.- A thin uniform rod of mass M and length L is positioned vertically above a frictionless pivot point, as shown in the figure, and then allowed to fall to the ground. With what angular velocity (ω) does the rod strike the ground?
Suggestion: Use conservation of mechanical energy to solve this problem
Moment of inertia of a rod rotating about one end = $\frac{1}{3}ML^2$



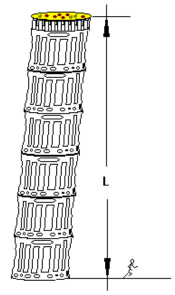
Solution: Using conservation of mechanical energy to solve this problem, notice that the potential energy of the rod is converted into rotational kinetic energy:

$$Mgh = \frac{1}{2} I\omega^2$$

The moment of inertia is given $I = \frac{1}{3}ML^2$, and h is L/2 because that is the location of the center of mass of the rod, so:

$$MgL/2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \rightarrow \omega = \sqrt{\frac{3g}{L}}$$

Problem 2.- The leaning tower of Pizza shown in the figure has a height of L=34 m and finally falls. Calculate the speed of the top of the tower when it hits the ground. To solve the problem consider the initial angle off the vertical to be negligible, assume that the mass of the tower is uniformly distributed, ignore any external torque and approximate the moment of inertia to the one of a rod rotated about one end $I = \frac{1}{3}ML^2$



Solution: We can solve this problem by considering all the potential energy being converted into rotational kinetic energy:

$$\mathrm{mgh} = \frac{1}{2}\mathrm{I}\omega^2$$

But there is an important detail, the center of mass of the tower is located at only half the height (at L/2), then:

$$\mathrm{mg}\frac{\mathrm{L}}{2} = \frac{1}{2}\left(\frac{1}{3}\mathrm{m}\times\mathrm{L}^2\right)\omega^2 \rightarrow \omega = \sqrt{\frac{3\mathrm{g}}{\mathrm{L}}}$$

And to find the speed:

$$\mathrm{v} = \omega\mathrm{r} = \sqrt{\frac{3\mathrm{g}}{\mathrm{L}}}\mathrm{L} = \sqrt{3\mathrm{gL}} = \sqrt{3\times9.8\times34} = \mathbf{32\ m/s}$$

Problem 3.- A clinical centrifuge accelerates uniformly from initial velocity zero to 3100 rpm in 6.0 seconds. Calculate the number of turns done by the rotor in that time.

Solution: The initial velocity is zero: $\omega_1 = 0$

The final velocity is 3100 rpm or in rad/s:

$$\omega_2 = 3100\frac{rev}{\min}\left(\frac{1\min}{60s}\right)\left(\frac{2\pi rad}{1rev}\right) = 324\ \text{rad/s}$$

The average velocity is: $\langle\omega\rangle = \dfrac{\omega_1 + \omega_2}{2} = \dfrac{0+324}{2} = 162 rad/s$

The angle is: $\theta = \langle\omega\rangle t = 162\times6 = 972$ rad

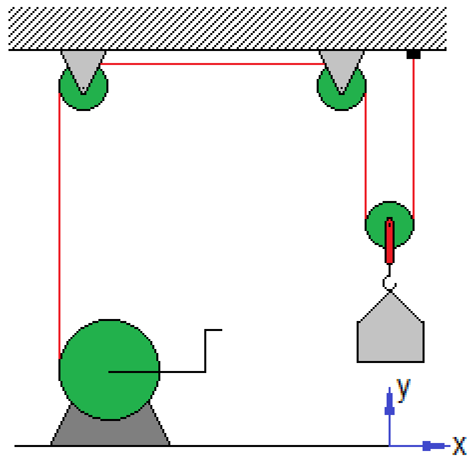
In turns: $\theta = 972 rad\left(\dfrac{1turn}{2\pi rad}\right) = \mathbf{155\ turns}$

Problem 4.- An engine accelerates uniformly from initial velocity 2000 rpm to 3200 rpm in 6.0 seconds. Calculate the number of turns done by the rotor in that time.

Solution: The average angular velocity is $\langle\omega\rangle = \dfrac{\omega_2 + \omega_1}{2} = 2600 rpm$ and the angle can be calculated simply by multiplying by time. Notice that 6 seconds is 0.1 minutes, so:

$$\theta = \langle \omega \rangle t = 2600 \frac{rev}{\min} \times 0.1 \min = \textbf{260 revolutions}$$

Problem 5.- A block is raised at a constant velocity of magnitude v. The three small pulleys have radius r and the large one R. The length of the handle is 2R. Take a coordinate system with x-axis to the right and y-axis upwards.



a) Determine the instantaneous position of the mass if at time t=0 its position was the origin of coordinates.

b) Determine the instantaneous angle of the large pulley.

c) How many turns do you need in the large pulley, so the mass reaches a height h?

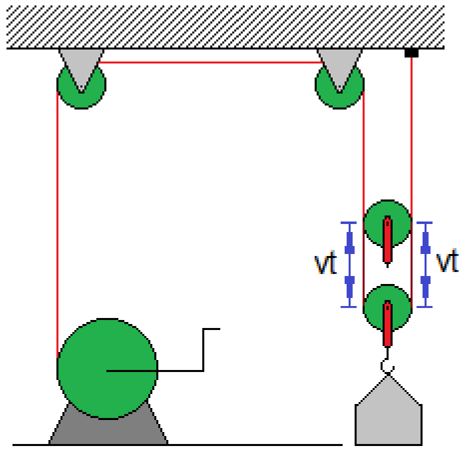
Solution:

a) Since the velocity is constant, the position is:

r = (0, vt, 0)

Here we have included a third component (z). The z-axis points out from the figure.

b) To find the angle, we notice that if the mass rises a distance vt, the string needs to be shorten by 2vt. To realize this, notice that there are two strings, one on each side of the pulley above the mass.



Then, the angle will be:

$\Theta = 2vt/R$

Besides, we notice that when the block rises, the pulley rotates counterclockwise, which is the direction of vector +k. Then, as a vector:

$\boldsymbol{\Theta} = (0, 0, 2vt/R)$

c) When the handle makes one turn, the pulley will shorten the string by $2\pi R$ and the mass will rise a distance πR, then, the number of turns for it to rise h is **$n = h/\pi R$**.

Problem 6.- Why does the nitrogen molecule in the atmosphere has more heat capacity than argon atoms?

Solution: It is due to its ability to store energy in its rotation. N_2 can also store energy in its vibrations, but this degree of freedom does not contribute as much to the heat capacity at normal temperatures of the atmosphere.

Problem 7.- You have two pulleys A and B with the same mass and with approximately cylindrical shapes. If the radius of pulley A is twice that of B, and both rotate at 360 rpm, what can we say about their rotational kinetic energies?

Solution: The moment of inertia of pulley A is four times that of B. This is because the radius enters in the equation squared. Since the angular velocity is the same, the energy of pulley A is four times that of B.

Problem 8.- A communications satellite rotates over its own axis at 10rpm to maintain a uniform temperature (while being heated by the sun). The satellite extends a retractable antenna, which increases its moment of inertia by 10%. What will happen to the rotational kinetic energy of the satellite?

Solution: Since we are not applying an external torque, the product of moment of inertia times the angular velocity needs to stay constant (the angular momentum). If I_o increases 10% to $1.1I_o$, the angular velocity ω_o will decrease to $\omega_o/1.1$ to compensate. But kinetic energy is $0.5I\omega^2$, so the kinetic energy will decrease to 1/1.1 of its original value or 9.1% approximately.

Problem 9.- A clutch disk rotates freely at 300rpm when another identical disk, initially at rest joins it and end up rotating together at 150rpm. What can we say about the kinetic energy before and after?

Solution: Because the moment of inertia of the two disks is twice as much as one disk, the final angular velocity is half of the initial value, thus maintaining the same angular momentum. However, the kinetic energy will decrease because it is 1/2 the product of I times the ***square*** of the angular velocity. The new I is twice as much, but ω is only half, so 50% of the original energy is gone.