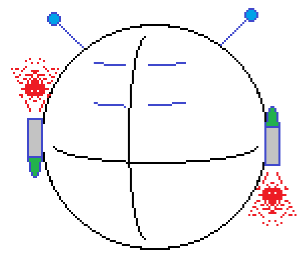
Physics I

Torque

Torque: $\tau = Fr\sin\angle_F^r$

Newton's second law for a rotating system: $\tau = I\alpha$

Problem 1.- Consider a spherical satellite of mass 5,500 kg and radius 2.5 m
Assume the mass of the satellite is uniformly distributed over the volume, so its moment of inertia is $\frac{2}{5}mR^2$
Two small rockets on the sides apply steady forces of 25N each to spin the satellite. Calculate how long we need to run the rockets to reach an angular velocity of 18rpm.



Solution: The initial angular velocity is zero, $\omega_1 = 0$

The final angular velocity will be $\omega_2 = 18\frac{rev}{\min}\left(\frac{1\min}{60s}\right)\left(\frac{2\pi}{1rev}\right) = 1.88 rad/s$

The angular acceleration is $\alpha = \frac{\omega_2 - \omega_1}{t}$

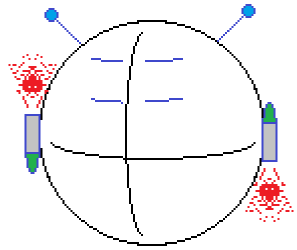
The total torque is $\tau = 2\times FR$, where the factor 2 is due to having two rockets. R is 2.5m and F is 25N.

The moment of inertia is $\frac{2}{5}mR^2$

Newton's second law for rotational systems is $\tau = I\alpha$, so in this case can be written as

$$2\times FR = \left(\frac{2}{5}mR^2\right)\frac{\omega_2 - \omega_1}{t} \rightarrow t = \frac{mR\omega_2}{5F} = \frac{5500\times 2.5\times 1.88}{5\times 25} = \mathbf{207\ s}$$

Problem 1a.- Consider a spherical satellite of 2.4m radius, whose mass is uniformly distributed close to its surface, so it can be modeled as a hollow sphere.
Two small rockets are turned on for 5 minutes and apply 15N each to make the satellite rotate from rest to 18rpm. Calculate the mass of the satellite. Ignore the mass lost by the rockets.



Solution: Using Newton's second law of motion (for rotation in this case) $\tau = I\alpha$
The torque is produced by the force of the rockets, which act a distance R from the center of rotation, so their total torque is $\tau = 2FR$

The moment of inertia of a hollow sphere is $I = \frac{2}{3}MR^2$

We also notice that the angular acceleration is $\alpha = \frac{\omega}{t}$

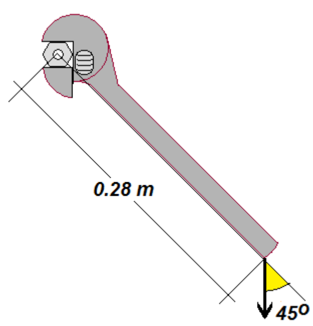
Substituting these equations in Newton's second law we get:

$$2FR = \frac{2}{3}MR^2 \frac{\omega}{t} \rightarrow M = \frac{3tF}{R\omega}$$

And with the values of the problem:

$$M = \frac{3(300s)(15N)}{2.4m(1.88rad/s)} = \mathbf{3{,}000\ kg}$$

Problem 2.- What should be the force F if the torque about the center of the nut should be 25 Nm?



Solution: $\tau = Fr\sin\angle_F^r$, so $F = \frac{\tau}{r\sin\angle_F^r}$

For a torque of 25 Nm: $F = \frac{25}{0.28\sin 45^\circ} = \mathbf{126\ N}$

Problem 3.- A ballplayer swings the bat reaching an angular velocity of 3.5rev/s in a time of 0.18s. Approximate the bat as a uniform rod of mass 2.6 kg and length 1.05m and calculate the torque applied by the athlete.

Moment of inertia of a rod rotating about one end is $I = \frac{1}{3}mL^2$

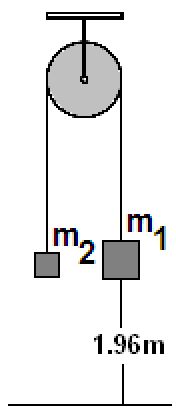
Solution: We can use Newton's second law to solve the problem

$$\tau = I\alpha \rightarrow \tau = \left(\frac{1}{3}mL^2\right)\left(\frac{\omega_2 - \omega_1}{t}\right)$$

Remember to change rev/s to rad/s: $\omega_2 = 3.5\frac{rev}{s}\left(\frac{2\pi\ radians}{1\ rev}\right) = 22\frac{rad}{s}$

$$\tau = \left(\frac{1}{3}mL^2\right)\left(\frac{\omega_2 - \omega_1}{t}\right) = \left(\frac{1}{3}\times 2.6\times 1.05^2\right)\left(\frac{22-0}{0.18}\right) = \mathbf{120\ Nm}$$

Problem 4.- In the Atwood machine shown in the figure m_1=10kg and m_2=9kg and the mass of the pulley is 2kg. Approximate the pulley as a disk ($I = \frac{1}{2}mR^2$). Ignore friction in the axis of the pulley. Find how long it will take for m_1 to hit the ground if you release the masses with zero initial velocity.



Solution: We find the acceleration first. To do this, consider that the change in potential energy is the same as the change in kinetic energy, so:

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 = (m_1 - m_2)gx$$

In this equation x is the distance dropped by mass 1. Now we take a derivative with respect to time and replace moment of inertia with $I = \frac{1}{2}mR^2$

$$m_1va + m_2va + I\omega\alpha = (m_1 - m_2)gv$$

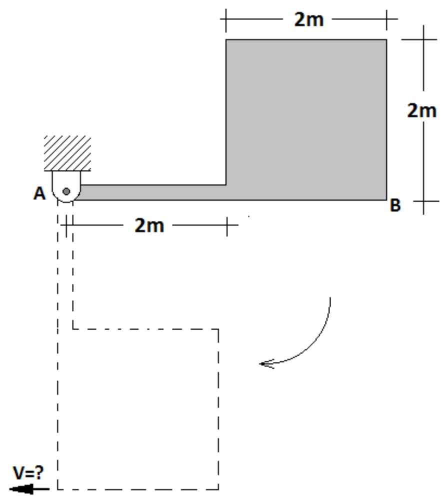
$$m_1va + m_2va + \frac{1}{2}mR^2\omega\alpha = (m_1 - m_2)gv$$

$$m_1 a + m_2 a + \frac{ma}{2} = (m_1 - m_2) g$$

The acceleration is: $a = \dfrac{(m_1 - m_2)}{m_1 + m_2 + \dfrac{m}{2}} g = \dfrac{(10-9)}{10+9+\dfrac{2}{2}} \times 9.8 = 0.49 m/s^2$

Now we can find the time: $x = \frac{1}{2} a t^2 \rightarrow t = \sqrt{\dfrac{2x}{a}} = \sqrt{\dfrac{2\times 1.96}{0.49}} =$ **2.83 s**

Problem 5.- Consider a mechanical piece built by joining a 2m long bar of negligible mass and a square plate of 2m side and mass 4kg. The piece is released from rest from the position shown below and it rotates freely around A. Calculate the velocity of point B when it passes the vertical line below A.



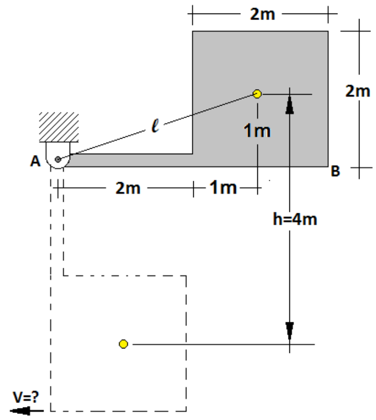
Solution: The mechanical piece will trade its potential energy for kinetic energy, which we can calculate with the equation $mgh = \frac{1}{2} I \omega^2$

As shown in the diagram below, the height h is the change in the center of mass, h=4m.
The moment of inertia is calculated using the parallel axes theorem. It is the sum of the moment of inertia with respect to the center of mass, plus the mass times the distance from the axis of rotation to the center of mass squared:

$$I = I_{CM} + m\ell^2$$

The value of ℓ is calculated by geometry and $I_{CM} = \frac{1}{6} m s^2$ is the moment of inertia of a square of side s and mass m rotating about its center of mass.

$$I = \frac{1}{6} \times 4 \left(2^2\right) + 4 \times (\sqrt{10})^2 = 42.66 kgm^2$$



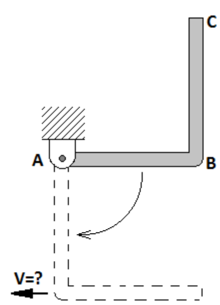
Substituting these values in the energy equation we obtain the angular velocity.

$$4(9.8)(4) = \frac{1}{2}(42.66)\omega^2 \rightarrow \omega = 2.711 rad/s$$

And to find the velocity of point B we multiply by the distance from B to the center of rotation.

$v = \omega r = 2.711 \times 4 =$ **10.8 m/s**

Problem 5a.- Consider a mechanical piece in the shape of an L, built by joining two identical thin 20cm long bars AB and BC. The piece is released from rest from the position shown below and it rotates freely around A. Calculate the velocity of point B when it passes the vertical line below A.



Solution: We use the same approach as the previous problem, based on the equation:

$$mgh = \frac{1}{2} I\omega^2$$

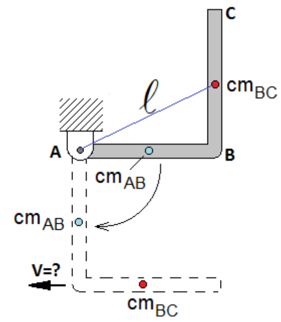
But, we divide the object in two pieces, so there are two potential energies and two kinetic energies:

$$mgh_{AB} + mgh_{BC} = \frac{1}{2} I_{AB}\omega^2 + \frac{1}{2} I_{BC}\omega^2$$

By observing the initial and final positions of the center of mass we find:

$h_{AB} = \frac{L}{2}$ and $h_{BC} = \frac{3L}{2}$

Where L is the length of the bars (20cm).



For AB the moment of inertia is simply the one of a rod rotated about one end:

$$I_{AB} = \frac{1}{3} mL^2$$

But for BC we need to consider the sum of the moment of inertia with respect to its center of mass plus the mass times the distance to the center of rotation squared (parallel axes theorem).

$$I_{BC} = \frac{1}{12} mL^2 + m\ell^2$$

Where $\ell^2 = \left(\frac{L}{2}\right)^2 + L^2 = \frac{5L^2}{4}$, so $I_{BC} = \frac{4mL^2}{3}$

Replacing these quantities in the energy equation:

$$mg\frac{L}{2} + mg\frac{3L}{2} = \frac{1}{2}\frac{mL^2}{3}\omega^2 + \frac{1}{2}\frac{4mL^2}{3}\omega^2$$

The angular velocity is

$$\omega = \sqrt{\frac{12g}{5L}}$$

And the speed of point B is

$$v_B = \omega L = \sqrt{\frac{12gL}{5}} = \sqrt{\frac{12 \times 9.8 \times 0.2}{5}} = \mathbf{2.17 m/s}$$