# Physics I

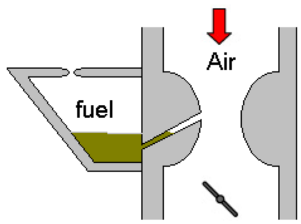
## Bernoulli

Definition of pressure: $P = \frac{\text{Force}}{\text{Area}}$

Hydrostatics equation: $P_B - P_A = \rho g h$

Bernoulli's equation: $P_1 + \frac{1}{2}\rho v_1^{\,2} + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^{\,2} + \rho g h_2$

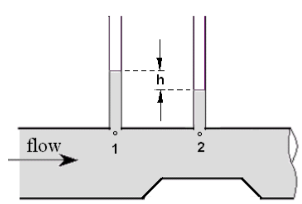
**Problem 1.-** In a carburetor (schematically shown in the figure) calculate the minimum speed of the air at the nozzle so the difference in pressure with the fuel reservoir is at least 1,500 pascals. [Take the density of air as 1.29 kg/m$^3$]




**Solution:** Using Bernoulli's equation, we get: $\Delta p = \rho \frac{v^2}{2} \rightarrow v = \sqrt{\frac{2\Delta p}{\rho}}$

And with the values of the problem: $v = \sqrt{\frac{2\times 1{,}500}{1.29}} =$ **48.2 m/s**

**Problem 2.-** The pipe in the figure is transporting oil (density 850 kg/m$^3$). The velocity at point 1 is 5m/s, but at point 2 it is 10m/s. Calculate the difference in height in the two open thin tubes.




**Solution:** First, we can calculate the pressure difference in pascals between points 1&2. Using Bernoulli's equation:

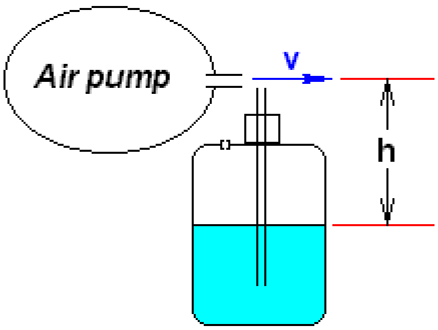
$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^{\,2}}{2g} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^{\,2}}{2g} \rightarrow \frac{P_1}{\rho g}{}_1 + \frac{v_1^{\,2}}{2g} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^{\,2}}{2g}$$

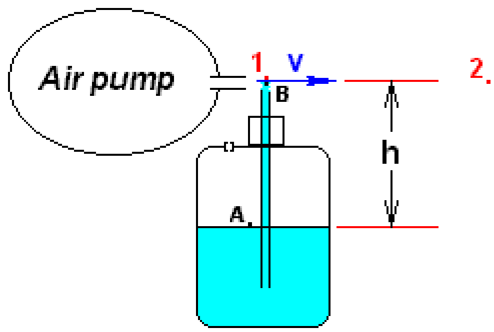
$$P_1 - P_2 = \rho \frac{v_2^{\,2} - v_1^{\,2}}{2} = 850 \times \frac{10^2 - 5^2}{2} = 31{,}875 \text{ pascals}$$

And now we use hydrostatics to find h:

$$P_1 - P_2 = 31{,}875 = \rho g h \rightarrow h = \frac{31{,}875}{850 \times 9.8} = \mathbf{3.83\ m}$$

**Problem 3.-** Perfume in a bottle has a density of 955 kg/m$^3$ and its level is h=0.025m below the nozzle as shown in the figure. Calculate the minimum speed of the air, so the liquid will reach the nozzle. [For the density of air use $\rho_{air} = 1.29 kg/m^3$]




**Solution:** The difference in pressure needed for the liquid to reach the nozzle can be calculated using the hydrostatic equation:

$$P_A - P_B = \rho g h = 955 \times 9.8 \times 0.025 = 234 \text{pascals}$$

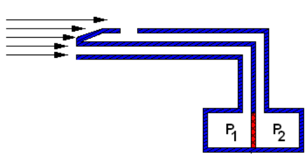
Now, to find the velocity that produces this change in pressure we need to compare points 1 and 2:

$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g}$$

However, the height is the same for points 1 and 2 and the velocity at point 2 is zero (far from the nozzle), so:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} \rightarrow \frac{v_1^2}{2} = \frac{P_2 - P_1}{\rho} \rightarrow v_1 = \sqrt{2 \times \frac{P_2 - P_1}{\rho}} = \sqrt{2 \times \frac{234}{1.29}} = \mathbf{19\ m/s}$$

**Problem 4.-** A Pitot tube is an instrument used to measure airspeed of an aircraft. Calculate the change in pressure read by the instrument if the airspeed is 105 m/s. [take the density of air at these conditions as 0.95kg/m$^3$]

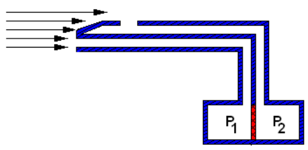


**Solution:** Using Bernoulli's equation: $\frac{P_1}{\rho g}+h_1+\frac{v_1^2}{2g}=\frac{P_2}{\rho g}+h_2+\frac{v_2^2}{2g}$ we get

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}=\frac{P_2}{\rho g}\rightarrow P_2-P_1=\rho\frac{v_1^2}{2}$$

If v=105m/s and $\rho=0.95kg/m^3$ then $P_2-P_1=0.95\frac{105^2}{2}=$ **5,200 pascals**

**Problem 4a.-** A Pitot tube is an instrument used to measure airspeed of an aircraft. Calculate the speed if the pressure difference read by the instrument is 7,500 pascals. Take the density of air at these conditions as 0.95kg/m$^3$.

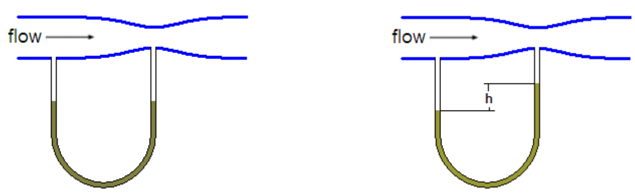



**Solution:** Using Bernoulli's equation: $\frac{P_1}{\rho g}+h_1+\frac{v_1^2}{2g}=\frac{P_2}{\rho g}+h_2+\frac{v_2^2}{2g}$ we get

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}=\frac{P_2}{\rho g}\rightarrow P_2-P_1=\rho\frac{v_1^2}{2}$$

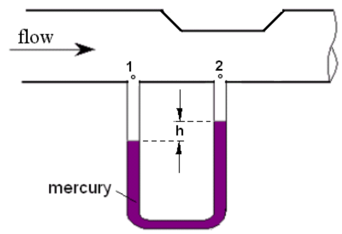
then $7500=0.95\frac{v^2}{2}\rightarrow v=$ **125 m/s**

**Problem 5.-** The figure shows a so called "Venturi tube" which is used to measure gas flow. The U-shaped tube section contains mercury, and the levels are equal because there is no flow right now. Use your knowledge of Bernoulli's principle to predict what will happen to the mercury levels when the gas flow starts.




**Solution:** The fluid will flow faster in the small cross section area, so the pressure will drop at that point. The mercury inside the tube will rise on the right and drop on the left.

**Problem 5a:** The Venturi tube shown in the figure has a restriction in the cross section, so the speed of the air flow at point "2" is 20 m/s, while the speed at point "1" is 10 m/s.
Calculate the difference in the level of mercury under these conditions.
[Take the density of air=1.29kg/m$^3$][1 torr = 133 pascals]




**Solution:** First, we can calculate the pressure difference in pascals and then convert to torr. Using Bernoulli's equation:

$$\frac{P_1}{\rho g}+h_1+\frac{v_1^{\,2}}{2g}=\frac{P_2}{\rho g}+h_2+\frac{v_2^{\,2}}{2g}\rightarrow\frac{P_1}{\rho g_{1}}+\frac{v_1^{\,2}}{2g}=\frac{P_2}{\rho g}+h_2+\frac{v_2^{\,2}}{2g}$$

$$P_1-P_2=\rho\frac{v_2^{\,2}-v_1^{\,2}}{2}=1.29\times\frac{20^2-10^2}{2}=194\text{ pascals}$$

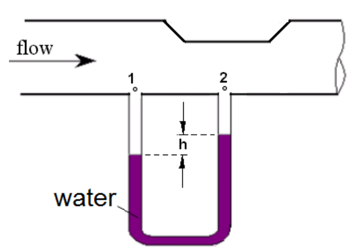
Converting to torr: $P_1-P_2=194\,pascals\left(\frac{1torr}{133\,pascals}\right)=1.45\text{ torr}$

So, **h=1.45 mm**

**Problem 5b.-** The Venturi tube shown in the figure has a restriction in the cross section, so the speed of the air flow at point "2" is 15 m/s, while the speed at point "1" is 10 m/s.

Calculate the difference in the level of water in the U-tube under these conditions.

[Take the density of air=1.29kg/m$^3$ and water=1000kg/m$^3$]




**Solution:** First, we can calculate the pressure difference in pascals. Using Bernoulli's equation:

$$\frac{P_1}{\rho g}+h_1+\frac{v_1^{\,2}}{2g}=\frac{P_2}{\rho g}+h_2+\frac{v_2^{\,2}}{2g}\rightarrow\frac{P_1}{\rho g_{1}}+\frac{v_1^{\,2}}{2g}=\frac{P_2}{\rho g}+h_2+\frac{v_2^{\,2}}{2g}$$

$$P_1 - P_2 = \rho \frac{v_2{}^2 - v_1{}^2}{2} = 1.29 \times \frac{15^2 - 10^2}{2} = 80.6 \text{ pascals}$$

Now we can find the value of h

$$P_1 - P_2 = \rho_{water} g h \rightarrow 80.6 = 1000 \times 9.8 \times h \rightarrow h = 0.0082 \text{ m}$$

So, **h=8.2 mm**

**Problem 6:** The airspeed on the top surface of a wing is 105m/s, but only 95m/s on the bottom surface. The wing has an area of 15m$^2$. Use Bernoulli's principle to calculate the net force trying to lift the wing. Ignore other mechanical effects such as viscosity drag.

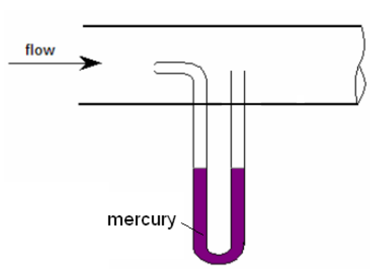
Take the density of air as 1.20kg/m$^3$

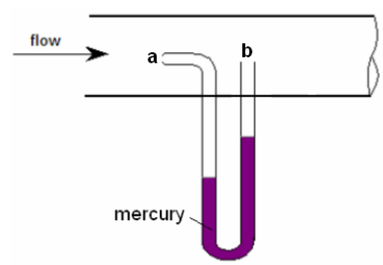
**Solution:** Bernoulli's principle: $\frac{1}{2}\rho v^2 + P + \rho g h = \text{constant}$

says that you can trade speed for pressure. When the wind blows above a surface the pressure drops, so there will be a net force given by the area of the wing times the difference in pressure:

$\text{Force} = (\text{Area})\frac{1}{2}\rho\left(v_1{}^2 - v_2{}^2\right)$ so, the force is: $F = (15)\frac{1}{2}(1.20)(105^2 - 95^2) =$ **18,000 N**

**Problem 7.-** A Pitot tube is an instrument used to measure airspeed of an aircraft or fluid flow in pipes. In the following schematic, the mercury is leveled because there is no flow. Indicate what will happen to the mercury when flow starts and give a short rationale of your answer.

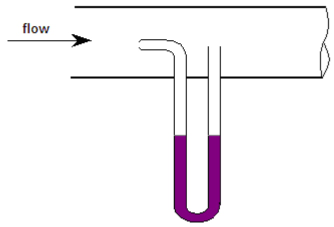


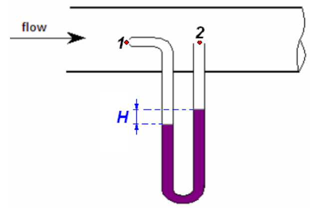




**Solution:** The average speed of molecules that are at point "a" is zero (they cannot flow anywhere) while the ones a point "b" have nonzero average velocity, so the pressure at "a" is larger than at "b" and the mercury will move to the position shown.

**Problem 7a.-** A Pitot tube is an instrument used to measure airspeed of an aircraft or fluid flow in pipes. In the following schematic, the mercury is leveled because there is no airflow. When the flow starts, the level on the right goes up 0.5mm (and the level on the left goes down 0.5

mm). Calculate the speed of the air if the density of air is 1.29kg/m$^3$ and the density of mercury is 13,600 kg/m$^3$.




**Solution:** We write Bernoulli's equation for points 1 and 2: $\frac{v_1^{\ 2}}{2g}+\frac{P_1}{\rho g}+h_1=\frac{v_2^{\ 2}}{2g}+\frac{P_2}{\rho g}+h_2$

The velocity at point 1 is zero as discussed in class. The heights are the same ($h_1 = h_2$), so we can eliminate them from the equation leaving:

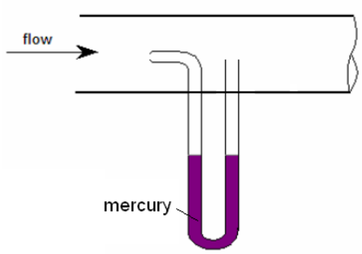
$$\frac{P_1}{\rho g}=\frac{v_2^{\ 2}}{2g}+\frac{P_2}{\rho g}\rightarrow\frac{P_1}{\rho}=\frac{v_2^{\ 2}}{2}+\frac{P_2}{\rho}\rightarrow v_2=\sqrt{\frac{2(P_1-P_2)}{\rho}}$$

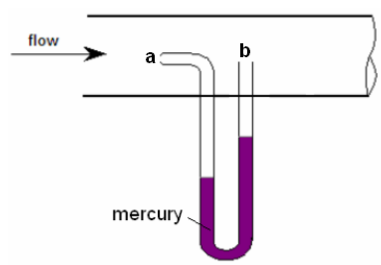
But the pressure difference can be calculated using the value of H:

$P_1 - P_2 = \rho_{Mercury} gH = 13{,}600 kg/m^3 (9.8 m/s^2)(1\times10^{-3} m) = 133.3$ pascals

So the velocity of the air is: $v_2=\sqrt{\frac{2(133.3\,pascals)}{1.29 kg/m^3}}=$ **14.4 m/s**

**Problem 7b:** A Pitot tube is an instrument used to measure airspeed of an aircraft or fluid flow in pipes. In the following schematic, the mercury is initially leveled because there is no flow. Calculate the difference in level when the speed of the flow of air is v=10.5 m/s.
[Take the density of air=1.29kg/m$^3$] [1 torr = 133 pascals]



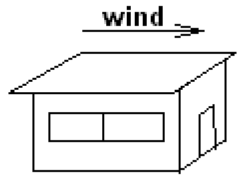

**Solution:** Using Bernoulli's equation: $\frac{P_a}{\rho}+gh_a+\frac{1}{2}v_a^2=\frac{P_b}{\rho}+gh_b+\frac{1}{2}v_b^2$

Considering that $v_a = 0$ and $h_a = h_b$ we get:

$$\frac{P_a}{\rho}=\frac{P_b}{\rho}+\frac{1}{2}v_b^2\rightarrow\frac{P_a-P_b}{\rho}=\frac{1}{2}v_b^2\rightarrow P_a-P_b=\frac{1}{2}\rho v_b^2=\frac{1}{2}(1.29)(10.5^2)=71 \text{ pascals}$$

In mm of mercury this is: **0.53 mmHg**

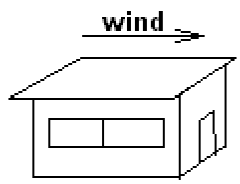
**Problem 8.-** How would you use Bernoulli's principle to calculate the force on a flat roof produced by wind of speed v? Write the equation(s) that you would use.




**Solution:** Bernoulli's equation can be used to calculate the change in pressure: $\frac{1}{2}\rho v^2 + P + \rho gh = \text{constant} \rightarrow \Delta P = \frac{1}{2}\rho v^2$ and then find the force by multiplying by area:

$$F = \frac{1}{2}\rho v^2 A$$

**Problem 8a.-** The wind is blowing at a speed of 25 miles/hour over a flat roof of area $95m^2$. Use Bernoulli's principle to calculate the net force trying to lift the roof.




**Solution:** Bernoulli's principle:

$$\frac{1}{2}\rho v^2 + P + \rho gh = \text{constant}$$

says that you can trade speed for pressure. When the wind blows above a roof the pressure drops outside, so there will be a net force given by the area of the roof times the difference in pressure:

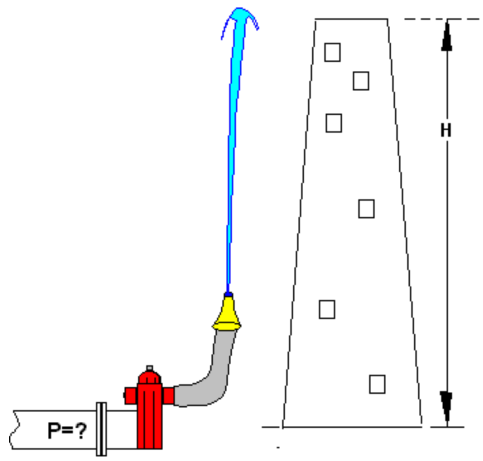
$$\text{Force} = (\text{Area})\frac{1}{2}\rho v^2$$

we need the speed in m/s:

$$v = 25\frac{\text{mile}}{\text{h}}\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{1609\text{m}}{1\text{mile}}\right) = 11.2\text{m/s}$$

So the force is: $\text{Force} = (95\text{m}^2)\frac{1}{2}(1.29\text{kg/m}^3)(11.2\text{m/s})^2 =$ **7,650 N**

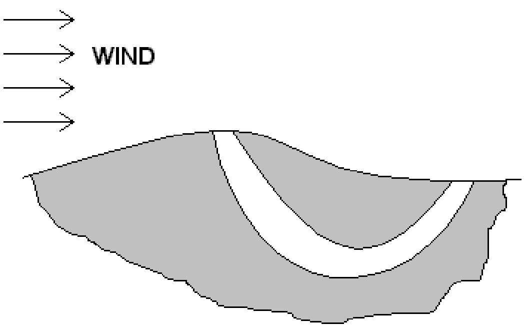
**Problem 9.-** You want water to reach a height of H=33 meters with a fire hose. Calculate the minimum gauge pressure in the mains to do this. Assume the speed of the water in the mains to be negligible and density $\rho_{water} = 1000\text{kg}/\text{m}^3$




**Solution**: Notice that the speed in the mains and the speed at the maximum height are both zero, so we can use the hydrostatic equation:

$P_A - P_B = \rho gh = 1000 \times 9.8 \times 33 =$ **323,000 pascals**

**Problem 10.-** Prairie dogs built the following architecture. Indicate in what direction air will flow in the tunnel… and explain **why**.




**Solution:** Bernoulli's equation $\frac{1}{2}\rho v^2 + P + \rho gh = \text{constant}$, says that you can trade speed for pressure. When the wind blows above the top of the mound, the pressure drops, so there will be a draft as shown below: