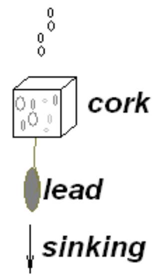
Physics I

Buoyancy

$F_{buoyancy} = \rho_{fluid}\, g Volume_{under\ surface}$ Buoyancy force

Problem 1.- A piece of cork of volume 0.000015m^3 and density $120 kg/m^3$ is floating on water. How much lead do you need to tie to the cube to sink it?
Density of lead is $11{,}300 kg/m^3$



Solution: The mass of cork is

$$mass = \rho Volume = 120\frac{kg}{m^3}(15\times10^{-6}m^3) = 0.0018\text{kg}$$

Let us assume that the mass of the lead is *M*. Then the volume of the lead will be:

$$Volume_{lead} = \frac{M}{11300 kg/m^3}$$

For the whole thing to sink, you need to have a buoyancy force equal to the weight:

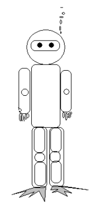
$$Weight = F_B = \rho_{water}\, g Volume$$

$$(M + 0.0018kg)g = 10^3\frac{kg}{m^3}(15\times10^{-6}m^3 + \frac{M}{11300 kg/m^3})g$$

Solving for M we get:

$$M + 0.0018kg = 0.015kg + \frac{M}{11.300} \rightarrow M = \frac{0.015kg - 0.0018kg}{1 - 1/11.3} = \mathbf{0.0146\ kg}$$

Problem 2.- You and your diving gear have a total mass of 110kg and a volume of 0.115m^3. How much lead (mass) do you need to carry in your belt to sink in seawater?
Take the density of seawater as 1,025 kg/m^3
Density of lead is 11,300 kg/m^3



Solution: The weight of the person with the gear and the lead must be the same as the buoyancy forces, so:

$$m_{diver}g + m_{lead}g = \rho_{sea-water}gV_{diver} + \rho_{sea-water}gV_{lead}$$

We can write the volume of the lead in terms of its mass: $V_{lead} = \dfrac{m_{lead}}{\rho_{lead}}$, so the equation becomes:

$$m_{diver}g + m_{lead}g = \rho_{sea-water}gV_{diver} + \rho_{sea-water}g\frac{m_{lead}}{\rho_{lead}}$$

And solving for the mass of lead we get:

$$m_{lead} = \frac{\rho_{sea-water}V_{diver} - m_{diver}}{1 - \dfrac{\rho_{sea-water}}{\rho_{lead}}} = \frac{1025 \times 0.115 - 110}{1 - 1025/11300} = \mathbf{8.66\ kg}$$

Problem 3.- Do ice cubes float higher or lower in an alcoholic drink (compared to pure water)? Why?
$[\rho_{\text{alcohol}} = 0.8\text{g/cm}^3 \qquad \rho_{\text{ice}} = 0.9\text{g/cm}^3 \qquad \rho_{\text{water}} = 1\text{g/cm}^3]$

Solution: Since the buoyancy force is equal to the product of density, volume and "g", the volume under the surface must be larger for an alcoholic drink, since its density is lower.
So, the **ice cubes will float lower**.

Problem 4.- What fraction of a block of wood (density=800kg/m^3) will be under the surface of mercury (density = 13,600 kg/m^3) when floating?

Solution: Let us assume a 1m^3 block of wood whose mass is 800 kg. To float, its weight should be equal to the buoyancy force:

$$mg = F_{buoyancy} = \rho_{fluid}\, g Volume_{under\ surface}$$

$$\rightarrow Volume_{under\ surface} = \frac{m}{\rho_{fluid}} = \frac{800kg}{13{,}600kg/m^3} = 0.0588\ \mathrm{m}^3$$

This is only **5.9 %** of the block's volume.

Problem 5.- The water of a very salty lake has a density of 1,220 kg/m3. Consider a 75-kg person floating in the lake. How much of her volume will be under the surface of the water?

Solution: The person will displace a volume of salty water that has the same weight as hers:

$$mg = \rho g Volume \rightarrow Volume = \frac{m}{\rho} = \frac{75kg}{1220kg/m^3} = \mathbf{0.0615\ m^3}$$

Problem 6.-

a) What is the buoyancy force due to air acting on a ping-pong ball of radius 20mm if the density of air is 1.29kg/m^3? [

The volume of a sphere is $\frac{4\pi R^3}{3}$

b) What fraction of its weight is the buoyancy force?

The mass of a ping-pong ball is 0.0027kg

Solution:

a) The buoyancy force:

$$F_B = \rho g V = (1.29kg/m^3)(9.8m/s^2)\frac{4\pi(0.0335m)^3}{3} = \mathbf{0.00199N}$$

b) The fraction of its weight that this represents:

$$\frac{F_B}{mg} = \frac{0.001991N}{0.0595kg(9.8m/s^2)} = \mathbf{0.34\%}$$