# Physics I

## Pressure and Density

$\text{Pressure} = \dfrac{\text{Force}}{\text{Area}}$ Definition of pressure. Measured in pascal (Pa) in SI units.

1atm=$1.013\times10^5$ Pa.

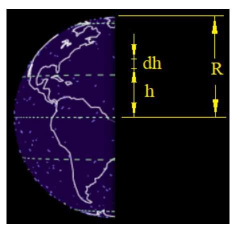
$\Delta P = \rho g h$ Hydrostatic pressure at a depth h.

**Problem 1.-** Estimate the pressure at the center of the Earth by integrating the hydrostatic equation ($\Delta P = \rho g h$) written in differential form:

$$dP = \rho g dh$$

So the integral will be:

$$P = \int_0^R \rho g dh$$




The limits of integration are from the center of the Earth ($h = 0$) to the surface ($h = R$).
Where $R = 6.37\times10^6$ m is the radius of the Earth.

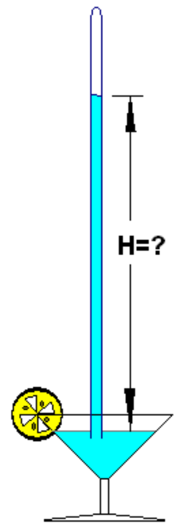
Take the density as a constant: $\rho = 5{,}500 \dfrac{\text{kg}}{\text{m}^3}$.

Take g to be this function: $g = 9.8 \dfrac{\text{m}}{\text{s}^2}\dfrac{h}{R}$.

**Solution:** $P = \int_0^R \rho g dh = \int_0^R 5500 \times \dfrac{9.8}{R} h dh = 5500 \times \dfrac{9.8 \times 6.37 \times 10^6}{2} =$ **172 GPa**

This rough calculation underestimates the real pressure because the density of the Earth is not constant, but it gives the right order of magnitude.

**Problem 2.-** How high would be the level in a barometer at normal atmospheric pressure if the fluid used were vodka martini (shaken not stirred) of density 910 kg/m$^3$? Would it be a practical instrument (why or why not)?




**Solution**: The pressure of the column of fluid inside the barometer has to match the atmospheric pressure, so:

$1atm = \rho gh$, which means that: $h = \frac{1atm}{\rho g} = \frac{1.013\times10^5 Pa}{\left(910\frac{kg}{m^3}\right)(9.8m/s^2)} = \mathbf{11.4m}$

This would not be a practical instrument, because it is too long and the vapor pressure of the liquid is too high.

**Problem 3.-**

a) Which has more volume: a kilogram of aluminum or a kilogram of gold?

b) Aerogel is a new material with very special properties. The silicon variety has a density of only $\rho = 1.1\text{mg/cm}^3$. Calculate the mass in grams of a 1.5-liter sample.

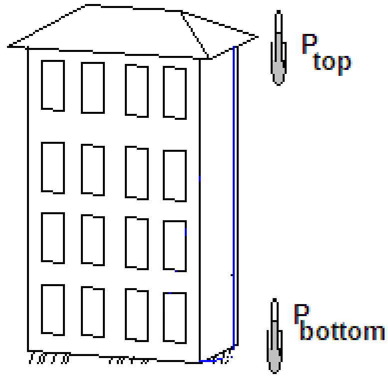
1 L=1000cm$^3$

**Solution:**

a) A kilogram of aluminum has more volume than a kilogram of gold because it has less density.

b) The mass of that sample is: $\text{m} = \rho\text{V} = (1.1\text{mg/cm}^3)(1.5\times10^3 cm^3) = \mathbf{1.65g}$

**Problem 4.-** A precise barometer is used to measure the height of a building. It gives a change in pressure of 172 pascal between the top and the bottom of the building shown in the figure. Assume the density of air is constant and equal to 1.25kg/m$^3$ to find "h".



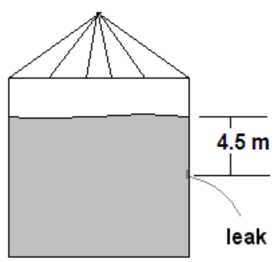

**Solution:** The change in pressure is $\Delta P = \rho g h$, so $h = \dfrac{\Delta P}{\rho g} = \dfrac{172\,pascals}{(1.25kg/m^3)(9.8m/s^2)}$ = **14m**

**Problem 5.-** If the density of air were constant 1.29kg/m$^3$ how high would be the atmosphere?

**Solution:** If the density were constant we could write:

$$1atm = \rho g h \rightarrow h = \frac{1atm}{\rho g} = \frac{1.013\times10^5\,pascals}{1.29kg/m^3(9.8m/s^2)}$$ = **8,010 m**

**Problem 6.-** A syrup tank springs a leak through a hole located 4.5m under the surface of the fluid. The area of the hole is 0.0035m$^2$. If you plug the hole with a rubber stopper, how much force must the rubber apply to stop the leak? [$\rho_{syrup} = 1{,}150 kg/m^3$]



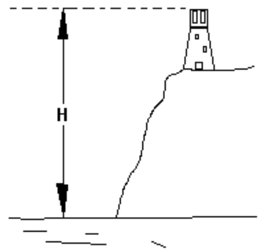

**Solution:** The change in pressure is

$$\Delta P = \rho g h = (1{,}150kg/m^3)(9.8m/s^2)(4.5m) = 50{,}715 \text{ pascal}$$

To find the force we multiply by the area:

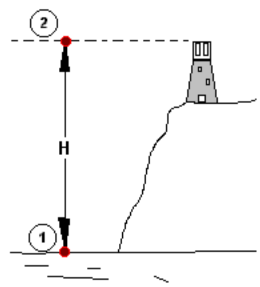
$F = (\Delta P)A =$ (50,715 pascal)(0.0035 m$^2$) = **177 N**

**Problem 7.-** Estimate the atmospheric pressure at an altitude H=140 meters over sea level, knowing that the pressure at sea level is $1.013\times10^{5}$ pascal.
In your estimation assume the density of air to be constant $\rho_{air} = 1.29 kg/m^{3}$.




**Solution:** For this estimation we use the hydrostatic equation:

$P \; - P_2 = \rho gh \rightarrow P_2 = P_1 - \rho gh = 1.013\times10^{5} - 1.29\times9.8\times140 =$ **99,500 pascal**




**Problem 8.-** Knowing that atmospheric pressure follows approximately the equation:

$P = 760e^{-h/8010}$

Where P is in torr and h is in meters. Calculate the height necessary to reach a pressure of 1 torr.

**Solution:** From the equation $P = 760e^{-h/8010}$ we get:

$1 = 760e^{-h/8010} \rightarrow h = -8010\ln\left(\frac{1}{760}\right) =$ **53,000 m**

**Problem 9.-** Calculate the pressure at the bottom of the Dead Sea, given that its depth is 330 m, the density of its salty water is 1230 $kg/m^3$ and the atmospheric pressure at the surface is $1.067\times10^{5}$ Pa.

**Solution:** From the equation $\Delta P = \rho gh$ we get:

$\Delta P = P_{bottom} - P_{surface} = \rho gh$

$P_{bottom} = P_{surface} + \rho gh =$ $1.033\times10^{5}$ Pa + (1,230 $kg/m^3$)(9.8$m/s^2$)(330m)= **4’081,000 Pa**

**Problem 9a.-** Calculate the pressure at the bottom of Lake Erie, whose depth is 64m. Assume the density of the water is 1,010 kg/m$^3$ and the atmospheric pressure at the surface is $1.033\times10^5$ Pa.

**Solution:**

P=$1.033\times10^5$ Pa+ $\rho gh$ = $1.033\times10^5$ Pa + (1,010 kg/m$^3$)(9.8m/s$^2$)(64m)= **737,000 Pa**

**Problem 10.-** Calculate the net force acting on a submarine window if the water depth is 200m and the area of the window is 0.025 m$^2$. Consider the pressure inside the submarine to be 1 atm. Take the density of seawater as 1,025 kg/m$^3$

**Solution:** The hydrostatic pressure at that depth is:

$$\Delta \mathrm{P} = \rho \mathrm{gh} = 1{,}025\mathrm{kg/m^3}(9.8m/s^2)(200m) = 2{,}000{,}000 Pa$$

The net force on the window will be:

$$F = Area\Delta \mathrm{P} = (0.025m^2)2{,}000{,}000Pa = \mathbf{50{,}000\ N}$$

**Problem 11.-** The gauge pressure in each of the four tires of an SUV is 28psi. Calculate the mass of the car if the "footprint" of each tire is 0.025m$^2$.

**Solution**: Let's convert the pressure in psi to pressure in pascal:

$$p = 28psi\left(\frac{6.9\times10^3 Pa}{1psi}\right) = 193{,}200 \text{ Pa}$$

By definition Pressure=$\frac{Force}{Area}$

So the force is just (Pressure)×(Area), but we need to multiply by 4 because there are four tires.

$$Force = 4PA = 4(193{,}200Pa)(0.025m^2) = 19{,}300 \text{ N}$$

To get the mass we divide the weight by g=9.8m/s$^2$

$$mass = \frac{19{,}300N}{9.8m/s^2} = \mathbf{1{,}970\ kg}$$

**Problem 12.-** To answer the two questions that follow take the density of gold and alcohol to be:

$\rho_{Au} = 19{,}300 \text{kg/m}^3$ $\rho_{alcohol} = 800 \text{kg/m}^3$

a) What is the volume of 1kg of alcohol?

b) What is the mass of 0.15 $m^3$ of gold?

**Solution**:

a) The volume of 1kg of alcohol: $\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{1\text{kg}}{800\text{kg}/\text{m}^3} = \mathbf{0.00125\ m^3}$

b) The mass of 0.15 $m^3$ of gold:

$$\text{Mass} = \text{density} \times \text{volume} = 19{,}300\text{kg}/\text{m}^3 \times 0.15\text{m}^3 = \mathbf{2{,}900\ kg}$$

**Problem 13.-** What is the difference in blood pressure between the feet and brain of a standing 1.8m-tall person? Approximate blood density to that of water.

**Solution**: $\Delta P = P_{feet} - P_{brain} = \rho g h = \frac{1{,}000 kg}{m^3} \frac{9.8 m}{s^2} 1.8 m = \mathbf{17{,}640\ Pa}$

This is 132 torr! So, it is no wonder that taking your blood pressure at the right height and position is important for accurate diagnoses.