Physics I

Viscosity

Viscosity: $\frac{F}{A} = \eta \frac{v}{l}$ for two parallel surfaces

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} \qquad \text{Poiseuille's equation}$$

Problem 1.- A pipeline has a diameter of 25.4cm and a difference in pressure of 30 psi. What new diameter would you need to increase the flow 3 times?

Solution: since the flow is proportional to R^4 we need to increase the diameter by a factor of $\sqrt[4]{3} = 1.32$, so the new diameter is 25.4×1.32 = **33.4 cm**

Problem 1a.- A pipeline has a diameter of 25.4cm (10 inches), but it is going to be replaced to accommodate 5 times the flow of oil. If you keep the same pressure difference, how much should be the new diameter?

Solution: With the same logic as the previous problem, if the flow is 5 times as large the diameter will need to be:

$$D = 25.4cm \times \sqrt[4]{5} = \mathbf{38\ cm}$$

Problem 2.- Based on the Poiseuille's equation for laminar flow with viscosity η, what must be the pressure difference between the two ends of a 19km pipeline, 12.3 cm in diameter if it is to transport oil at a rate of Q = 950cm^3/s?

The viscosity of oil is $\eta = 0.20\ \text{Pa} \cdot \text{s}$

Solution: Poiseuille's equation for laminar flow:

$$Q = \frac{\text{volume}}{\text{time}} = \frac{\pi P R^4}{8 L \eta}$$

So, solving for the pressure: $P = \frac{8L\eta}{\pi R^4} Q$

Where: L=19km=19,000 m

R=12.3cm/2=0.0615m

$$Q = \frac{950\text{cm}^3}{\text{s}} = 950 \times 10^{-6}\,\text{m}^3/\text{s}$$

Then: $P = \frac{8L\eta}{\pi R^4} Q = \frac{8 \times 19000 \times 0.2}{\pi \times 0.0615^4} \times 950 \times 10^{-6} = \mathbf{643{,}000\ Pa}$

Problem 3.- Based on the Poiseuille's equation for laminar flow

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}$$

Suppose the radius of an artery is reduced to 0.8R due to accumulation of plaque. By what factor do you need to increase the pressure difference $(P_1 - P_2)$ to keep the same flow?

Solution: Since the radius is only 0.8 times the old radius the flow would be $0.8^4 = 0.4096$ of the old flow. To compensate for this, we need to increase the pressure difference by a factor of 1/0.4096=**2.44**