Physics I

Doppler Effect

For a detector moving directly towards a stationary emitter use the + sign (− if moving away).

$$f_{\text{detected}} = f_{\text{emitter}}\left(1 \pm \frac{v_{\text{detector}}}{v_{\text{sound}}}\right)$$

For an emitter moving directly towards a stationary detector use the − sign (+ if moving away).

$$f_{\text{detected}} = \frac{f_{\text{emitter}}}{\left(1 \mp \frac{v_{\text{emitter}}}{v_{\text{sound}}}\right)}$$

For electromagnetic waves, if the detector is directly approaching the emitter (or vice versa).

$$f_{\text{detected}} = f_{\text{emitter}}\sqrt{\frac{1+\beta}{1-\beta}} \qquad \text{where} \quad \beta = \frac{v}{c} = \frac{v}{3\times 10^{8}\,\text{m/s}}$$

If the detector is moving directly away from the emitter, you change the signs.

Problem 1.- A bat (flying mammal, not baseball) emits an ultrasonic wave of frequency 48.5 kHz while moving towards an ultrasonic microphone at 7 m/s. Calculate the frequency measured by the instrument.

Solution: Using the Doppler Effect equation for a stationary detector.

$$\text{f}_{\text{detected}} = \frac{\text{f}_{\text{emitter}}}{\left(1\text{-}\frac{\text{v}}{\text{v}_{\text{sound}}}\right)} = \frac{48{,}500\text{Hz}}{\left(1\text{-}\frac{7\text{m/s}}{343\text{m/s}}\right)} = \quad \mathbf{49{,}500\ Hz}$$

Problem 2.- A source of radio waves emits radiation of wavelength $\lambda = 0.21061$m Find the wavelength observed on Earth if the source approaches us at $v = 335\times10^{3}$ m/s.

Solution: The Doppler Effect indicates that the frequency will be higher, so the wavelength will be shorter, given by the equation:

$$\lambda_{\text{detected}} = \lambda_{\text{emitter}}\sqrt{\frac{1-\beta}{1+\beta}}$$

With the values of the problem we get:

$$\lambda_{\text{detected}} = 0.2106\sqrt{\frac{1-335\times10^{3}/3\times10^{8}}{1+335\times10^{3}/3\times10^{8}}} = \mathbf{0.21036\ m}$$

Problem 3.- Two speakers separated by 100m emit sound at a frequency of 1,500 Hz simultaneously. You are at the middle point between the two speakers and run towards one of them (and away from the other) at a speed of 5m/s. Calculate the frequency of the resulting beats. Take the speed of sound as 343m/s.

Solution: The detector (you) will hear a higher pitch coming from the source that you are approaching.

$$f_{detected} = f_{emitter}\left(1+\frac{v_{detector}}{v_{sound}}\right) = 1{,}500\left(1+\frac{5}{343}\right) = 1{,}522 \text{ Hz}$$

However, you will hear a lower pitch coming from the other source

$$f_{detected} = f_{emitter}\left(1-\frac{v_{detector}}{v_{sound}}\right) = 1500\left(1-\frac{5}{343}\right) = 1{,}478 \text{ Hz}$$

The beats will have a frequency of 1,522-1,478= **44 Hz**

Problem 4.- In studying a star you notice that the Hα spectral line (whose normal wavelength in the lab is 659.8 nm), is shifted to 658.4 nm.

(A) Is it approaching us or moving away from us?

(B) What is the speed of the star?

Solution: (A) since the detected wavelength is shorter, this is a case of blue-shift, where the star is approaching us.

(B) We use the Doppler Effect equation to solve for the speed:

$$\lambda_{detected} = \lambda_{emitter}\sqrt{\frac{1-\beta}{1+\beta}} \rightarrow \beta = \frac{\lambda_{emitter}^2 - \lambda_{detected}^2}{\lambda_{emitter}^2 + \lambda_{emitter}^2} \rightarrow v = \frac{\lambda_{emitter}^2 - \lambda_{detected}^2}{\lambda_{emitter}^2 + \lambda_{emitter}^2} c$$

With the values given:

$$v = \left(\frac{659.8^2 - 658.4^2}{659.8^2 + 658.4^2}\right) 3\times10^8 \frac{\text{m}}{\text{s}} = \mathbf{637\ km/s}$$

Alternatively, since v/c is so small, it is possible to approximate $\sqrt{\frac{1-v/c}{1+v/c}} \approx 1 - v/c$, so

$$\frac{\lambda_{detected}}{\lambda_{emitter}} = 1 - v/c \rightarrow v = \left(1 - \frac{\lambda_{detected}}{\lambda_{emitter}}\right) c = \left(1 - \frac{658.4}{659.8}\right) 3\times10^8 \frac{\text{m}}{\text{s}} = \mathbf{637\ km/s}$$

Problem 5.- At what frequency do you hear a police car siren that emits a frequency of 500Hz and is approaching you at 22m/s?

Solution: Since the siren is the emitter and it is moving, we should use the equation:

$$f_{detected} = f_{emitter} \frac{1}{1 - v/v_{sound}}$$

We use the negative sign because the source is approaching the detector (us):

$$f_{detected} = 500\text{Hz} \frac{1}{1 - 22/343} = \mathbf{534\ Hz}$$

Problem 6.- A bat at rest sends a sound wave with a frequency of 45.0 kHz and receives it returning from an object moving directly away from it with a frequency of 42.4 kHz. Find the speed of the object.
Speed of sound = 343m/s

Solution: This is another case of the Doppler Effect: a source that is not moving and an object that reflects the waves back to the source, where a detector measures the frequency. For an object moving directly away from the source the frequency is given by:

$$f_{detected} = f_{emitter} \frac{1 - \frac{v_{object}}{v_{sound}}}{1 + \frac{v_{object}}{v_{sound}}}$$

We know the two frequencies and the speed of sound. Some algebraic manipulation gives us:

$$v_{object} = v_{sound} \frac{1 - \frac{f_{detected}}{f_{emitter}}}{1 + \frac{f_{detected}}{f_{emitter}}} \rightarrow v_{object} = 343 \frac{1 - \frac{42.4}{45.0}}{1 + \frac{42.4}{45.0}} = \mathbf{10.2\ m/s}$$