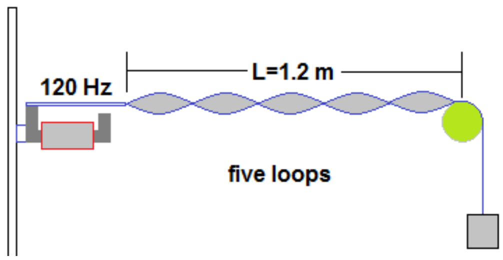
Physics I

Standing Waves

Speed of a standing wave $v = \sqrt{\frac{F_{tension}}{\rho_{linear}}}$

$f = \frac{n}{2L}\sqrt{\frac{F_T}{\rho_{linear}}}$ Standing waves on a string

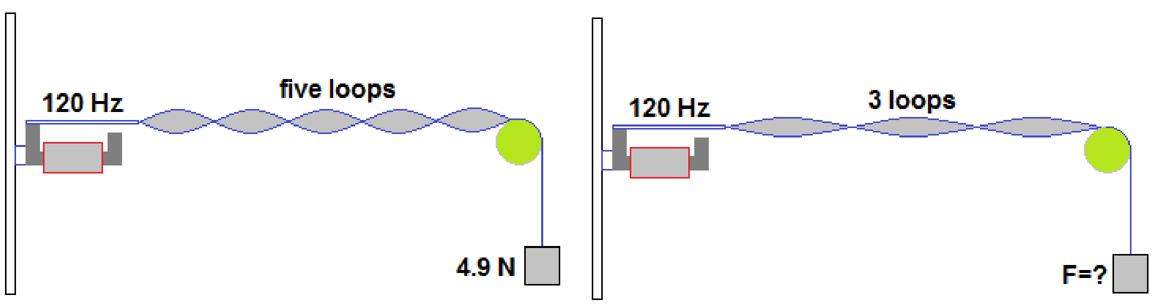
Problem 1.- Suppose you have a standing wave forming five loops with a vibrating magnet of frequency 120 Hz. If the length of the string is 1.2 m, calculate the speed of the wave.



Solution: Five loops have a length of 1.2 m, so each loop is 0.24 m long. A wavelength is two loops, so $\lambda = 0.48m$ and using the fundamental equation of waves:

$v_{wave} = \lambda f = 0.48 \times 120 =$ **58 m/s**

Problem 2.- Suppose you have a standing wave forming five loops. This is accomplished by putting a string under a tension F=4.9 N and a vibrating device with frequency 120Hz. How much tension do you need to get 3 loops?



Solution: Notice that in the equation $f = \frac{n}{2L}\sqrt{\frac{F}{\rho_{linear}}}$ the force is inversely proportional to the number of loops squared, so to 3 loops instead of 5 we should multiply the force by $5^2/3^2$=25/9, that means that instead of 4.9 N it will be **13.6 N**.

Problem 3.- A guitar string produces a fundamental frequency of 293Hz. Another string in the same guitar is under the same tension, but it has three times the linear density (it is thicker). What is the fundamental frequency of the second string?

Solution: We use the equation $f = \frac{n}{2L}\sqrt{\frac{F_{tension}}{\rho_{linear}}}$

In the first case $293 = \frac{1}{2L}\sqrt{\frac{F_{tension}}{\rho_{linear}}}$

In the second case $f = \frac{1}{2L}\sqrt{\frac{F_{tension}}{3\rho_{linear}}}$

Dividing the second equation by the first: $\frac{f}{293} = \frac{\frac{1}{2L}\sqrt{\frac{F_{tension}}{3\rho_{linear}}}}{\frac{1}{2L}\sqrt{\frac{F_{tension}}{\rho_{linear}}}} = \frac{1}{\sqrt{3}}$

So: $f = \frac{293}{\sqrt{3}} =$ **169 Hz**

Problem 4.- A guitar string is tuned so its fundamental frequency is 440Hz, but then you tighten it, so the tension increases 2%. What is the new frequency?

Solution: The wavelength of the string vibration will not change because the length stays the same, but increasing the tension will increase the speed of the wave in the string, so the frequency will increase as shown below:

We know that:

$$440Hz = \frac{\sqrt{\frac{F_T}{\rho_{linear}}}}{\lambda}$$

and the new frequency is:

$$f = \frac{\sqrt{\frac{1.02F_T}{\rho_{linear}}}}{\lambda} = \sqrt{1.02}\left(\frac{\sqrt{\frac{F_T}{\rho_{linear}}}}{\lambda}\right) = \sqrt{1.02}(440Hz) = \mathbf{444.4Hz}$$

Problem 5.- A violin string produces a sound whose fundamental frequency is 312 Hz when free to vibrate. You put your finger 1/3 of the way from the top, so only 2/3 of the length can vibrate. What is the new fundamental frequency?

Solution: Notice that in the equation $f = \frac{n}{2L}\sqrt{\frac{F}{\rho_{linear}}}$ if you reduce the length from L to 2/3 of L is the same as multiplying the frequency by 3/2, so instead of 312 Hz it will be **468 Hz**.

Problem 6.- A guitar string has a linear density of 3.5 g/m. The distance from the bridge to the support post is L=0.65 m and the string is under a tension of 250N. What is the frequency of the fifth harmonic?

Solution: This is a direct application of the equation $f = \frac{n}{2L}\sqrt{\frac{F_T}{\rho_{linear}}}$

The only consideration is that we should be careful using the right units:

n=5 because we want the fifth harmonic
L=0.65 m
F_T=250 N
$\rho_{linear} = 3.5g/m = 0.0035kg/m$

$$\text{So: } f = \frac{n}{2L}\sqrt{\frac{F_T}{\rho_{linear}}} = \frac{5}{2(0.65m)}\sqrt{\frac{250N}{0.0035kg/m}} = \mathbf{1{,}027\ Hz}$$

Problem 7.- A cord of mass 0.75kg is stretched between two supports 28m apart. If the tension in the cord is 250N, how long will it take a pulse to travel from one support to the other?

Solution: The speed of the pulse will be: $v = \sqrt{\frac{F_{tension}}{\rho_{LINEAR}}} = \sqrt{\frac{250N}{(0.75kg/28m)}} = 96.6m/s$

To find the time we divide the distance by the speed:

$$t = \frac{x}{V} = \frac{28m}{96.6m/s} = \mathbf{0.29\ s}$$

Problem 8.- A cable of mass 100 kg is stretched between two poles 115 m apart. Find the tension in the cable if it vibrates with a third harmonic of 1.35 Hz.

Solution: To solve this problem, we can apply what we learned about waves on a string. The third harmonic is 1.35 Hz, which means that the fundamental is one third of that frequency, or 0.45 Hz.
The linear density of the cable can be determined from the information provided (100kg, 115m) as follows:

$$\rho_{linear} = \frac{100kg}{115m}$$

We can use the equation that we deduced in class for the frequency and solve for the tension:

$$f = \frac{1}{2L}\sqrt{\frac{F_T}{\rho_{linear}}} \rightarrow f^2 = \frac{1}{4L^2}\frac{F_T}{\rho_{linear}} \rightarrow F_T = 4L^2 f^2 \rho_{linear}$$

$$\rightarrow F_T = 4(115m)^2(0.45Hz)^2\frac{100kg}{115m} = \mathbf{9{,}300\ N}$$