

Physics I

Dot and Cross Products

Problem 1.- What is the angle between the vectors $(-1,1)$ and $(3,4)$?

Solution:

$$\text{The angle is: } \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right) = \cos^{-1}\left(\frac{-3+4}{|5|\sqrt{2}}\right) = \cos^{-1}\left(\frac{1}{5\sqrt{2}}\right) = \mathbf{82^\circ}$$

Problem 2.- Find the angle between the vectors:

$$A = (1, 2, 3)$$

$$B = (-1, 2, -3)$$

Solution: Notice that:

$$A \cdot B = -1 + 4 - 9 = -6 \text{ and:}$$

$$|A| = \sqrt{1+4+9} = \sqrt{14}$$

$$|B| = \sqrt{1+4+9} = \sqrt{14}$$

$$\text{So: } \cos \theta = \frac{A \cdot B}{|A||B|} = \frac{-6}{14} \rightarrow \theta = \cos^{-1}\left(\frac{-6}{14}\right) = \mathbf{115^\circ}$$

Problem 3.- Vector $A = (2,3,1)$ is perpendicular to the vector $B = (x, -2,1)$. Calculate the value of x .

Solution: The dot product must be zero if the vectors are perpendicular so,

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos \angle_A^B = 0 \rightarrow 2x - 6 + 1 = 0 \rightarrow x = \mathbf{2.5}$$