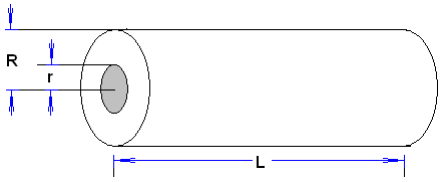
Physics I

Viscosity



The general form of the equation that describes viscosity is given by:

$$\frac{\mathrm{F}}{Area} = \eta \frac{dv}{dl}$$

Where *Area* corresponds to the layer of fluid with velocity v.

If we apply the equation to the velocity in the cylinder shown above, we get:

$$\frac{\mathrm{P}(\pi r^2)}{2\pi r L} = -\eta \frac{dv}{dr}$$

Notice that the pressure times the area of the base of the cylinder gives us the force. Also, notice that the layer of fluid has an area equal to the lateral area of the cylinder. Another subtlety is that the derivative has a minus sign in front.

Simplifying: $\frac{\mathrm{Pr}}{2L\eta} dr = -dv$ integrating: $v = \frac{P(R^2 - r^2)}{4L\eta}$

Now, we can calculate the volume per time:

$$Q = \frac{volume}{time} = \int_0^R v(2\pi r)dr = \int_0^R \frac{P(R^2 - r^2)}{4L\eta}(2\pi r)dr = \frac{\pi P R^4}{8L\eta}$$

This is Poiseuille's equation of flow rate in a cylindrical pipe.