Physics II

Gauss's Law

In electrostatics, if you know the position of all the electric charges, it is possible to calculate the electric field using Coulomb's law for each charge and adding (or integrating) the vectors. Alternatively, in cases with symmetry, it is possible to use Gauss's law to simplify the calculations.

Note: Something similar happens in magnetism. It is possible to use the law of Biot and Savart to calculate the magnetic field produced by electric currents, but in symmetric cases it is easier to use Ampere's law instead.

Gauss's law indicates that the total electric flux through a closed surface is equal to the total charge inside the surface divided by the electric permittivity.

$$\phi = \frac{Q}{\varepsilon_\circ}$$

In this equation ϕ is the flux through the surface. It is defined as the product of the surface are times the electric field component perpendicular to it.

$$\phi = Area(E_\perp)$$

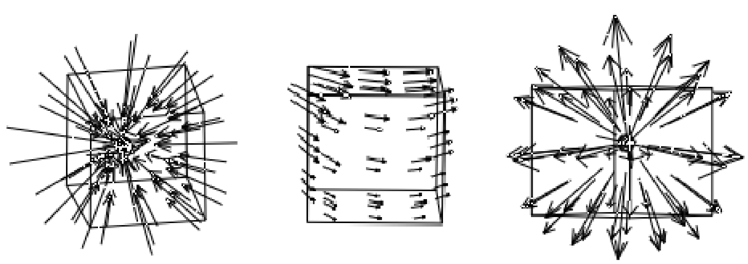
This electric field component is taken positive if the field point out from the surface and negative if it points inwards.

In case when the field is a function of position, it will be necessary to add the parts or integrate.

$$\phi = \sum_n Area_n (E_\perp)_n$$

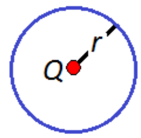
$$\phi = \oint_S E_\perp dA = \oint_S E.dA$$

The images show electric field vectors on the surface of a cube. First surrounding a negative charge, then in vacuum and then surrounding a positive charge.



Next we will see some simple cases where we can take advantage of the symmetry.

Case 1, point charge.- In this case we can use a spherical surface of radius r that surrounds the charge. Due to symmetry, we expect that the direction of the field will be radial, perpendicular to the surface.



Applying Gauss's law:

$$E(4\pi r^2) = \frac{Q}{\varepsilon_\circ}$$

Getting:

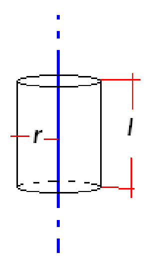
$$E = \frac{Q}{4\pi\varepsilon_\circ r^2}$$

We see that we obtain the well-known Coulomb's law.

We also notice that for any spherical distribution of charge it is equivalent to having all the charge in the center of the sphere.

Case 2, charged wire.- Consider a very long wire with uniform linear charge density that we call lambda (λ), in units of C/m. We want to calculate the electric field at a distance r from the wire.

We consider a cylindrical surface as shown below, which encloses a section of length *l* of the wire.



The enclosed charge is $Q = \lambda l$

Due to symmetry, there is no electric field parallel to the wire and because of that, the flux through the top and bottom circles will be zero. Instead through the lateral area of the cylinder we expect the field to be radial, so perpendicular to the surface. Gauss's law indicates that:

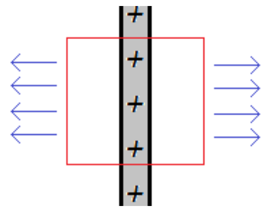
$$\mathrm{E}(2\pi \mathrm{r}\ell) = \frac{\lambda\ell}{\varepsilon_\circ}$$

The electric field is:

$$\mathrm{E} = \frac{\lambda}{2\pi\varepsilon_\circ \mathrm{r}}$$

It is interesting to notice that in this case the electric field decays like the inverse distance, not inverse squared. This result can be applied as an approximation for a transmission line.

Case 3, Flat uniformly charged surface.- Consider a large plane where the electric charge is uniformly distributed with a density per area that we will call sigma (σ) with units of C/m^2.



Consider a Gauss surface such as cube with side L that encloses an area L^2 of the surface with charge σL^2. Due to symmetry, we expect the electric field to be perpendicular to two sides of the cube and parallel to the other four. Then, we can write Gauss's law as:

$$\mathrm{E}(2L^2) = \frac{\sigma L^2}{\varepsilon_\circ}$$

And the electric field is:

$$\mathrm{E} = \frac{\sigma}{2\varepsilon_\circ}$$

You have to be careful that in this case the electric field points away from the surface on both sides. It is possible for the field to exist only on one side, for example in a parallel-plates capacitor. In that case the field will be $\mathrm{E} = \dfrac{\sigma}{\varepsilon_\circ}$

It is also interesting that the field is uniform and does not depend on the distance to the surface.