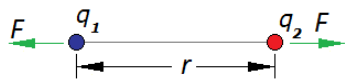
Physics II

Electric Field and Forces

Coulomb's Law

Experimentally it is observed that there is a force between electric charges, which is proportional to the magnitude of the charges and inversely proportional to the distance between them squared. The figure below shows this phenomenon.



This called Coulomb's law, and it is one of the fundamental equations of electromagnetism. We can write it as follows:

$$F = k\frac{q_1 q_2}{r^2}$$

F is the magnitude of the force. We measure it in newton, represented by the letter N.

q_1 and q_2 are the charges. They are measured in coulomb, represented by C.

r is the distance between the charges in meters (m).

k is a constant of nature that relates the force with the charges and distance, so its units are $\mathrm{Nm^2/C^2}$, which you can confirm with dimensional analysis. Its value is

$$k = 9\times 10^9\,\mathrm{Nm^2/C^2}$$

Sometimes it is convenient to express this constant in terms of another constant called $\varepsilon_\circ$ (epsilon naught) as follows:

$$k = \frac{1}{4\pi\varepsilon_\circ}$$

Epsilon naught is also called the dielectric constant or vacuum permittivity. You can derive its value from the definition above.

$$\varepsilon_\circ = 8.85\times 10^{-12}\,\mathrm{C^2/Nm^2}$$

If the charges have the same sign, the force will be repulsive, while if one is positive and the other negative, the force is attractive.

Let us examine one example.

Example 1.- Calculate the electric force between a proton and an electron separated by a distance of 1nm.

Solution: The proton has a positive charge equal to $q_1 = 1.60\times10^{-19}\text{C}$ and the electron has negative charge of the same magnitude $q_2 = -1.60\times10^{-19}\text{C}$. If the separation between them is 1nm then the force will be

$$\text{F} = \text{k}\frac{\text{q}_1\text{q}_2}{\text{r}^2} = 9\times10^9\text{Nm}^2/\text{C}^2\,\frac{(1.60\times10^{-19}C)(-1.60\times10^{-19}C)}{(1\times10^{-9}m)^2} = \mathbf{-2.3\times10^{-10}\ N}$$

The minus sign indicates that the force is attractive.

Comparison with gravitational force

The gravitational force is similar to the Coulomb force. It is also inversely proportional to the square of the distance and directly proportional to the product of the masses, but there are two differences:

a) The gravitational force is extremely weak.
The numerical value obtained in the previous example might seem small, but let us compare it to the gravitational force between the proton and the electron at the same distance.

$$\text{F}_{\text{gravitation}} = \text{G}\frac{\text{m}_1\text{m}_2}{\text{r}^2} = 6.67\times10^{-11}\text{Nm}^2/\text{kg}^2\,\frac{(1.67\times10^{-27}kg)(9.11\times10^{-31}kg)}{(1\times10^{-9}m)^2} = 1.01\times10^{-49}N$$

This value is 2,270,000,000,000,000,000,000,000,000,000,000,000,000 times smaller than the electric force.

b) The gravitational force is attractive only, while the electric force can be attractive or repulsive. Different from gravity where there is only one kind of mass, in electricity, charge can be positive or negative.

This is important in everyday life because in almost anything that surrounds us, the positive charge of the nucleus is completely equilibrated with the negative charge of the electrons giving a neutral total.

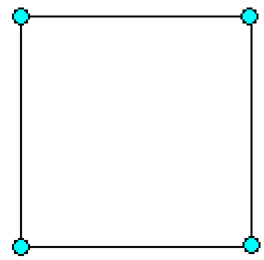
It is interesting to think that the most extraordinary electric forces that we see in nature, like storm lighting, are caused by very tiny unbalance between positive and negative charges.

Dealing with vectors

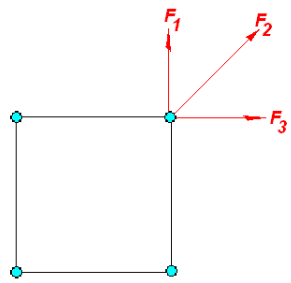
Recall that forces are vectors with magnitude and direction, so if we have to add several electric forces, we need to do this as vectors.

Next, an example of this case:

Example 2.- We place charges of magnitude q = 6μC at the corners of a square of side 0.35m. Determine the magnitude of the force on each charge.



Solution: Consider the forces on the charge located at the top right corner due to the other three charges.



$$F_1 = k\frac{q_1 q_2}{r^2} = 9\times10^9 \frac{(6\times10^{-6})(6\times10^{-6})}{0.35^2} = 2.64N$$

$$F_2 = k\frac{q_1 q_2}{r^2} = 9\times10^9 \frac{(6\times10^{-6})(6\times10^{-6})}{(0.35\sqrt{2})^2} = 1.32N$$

$$F_3 = k\frac{q_1 q_2}{r^2} = 9\times10^9 \frac{(6\times10^{-6})(6\times10^{-6})}{0.35^2} = 2.64N$$

All three forces are repulsive and have the directions shown in the figure. To sum them up we need to find their components and add them.

$$F_1 = (0,2.64)$$

$$F_2 = (1.32\cos 45^\circ, 1.32\sin 45^\circ) = (0.93,0.93)$$

$$\underline{F_3 = (2.64,0)}$$

$$\vec{F} = (3.57,3.57)N$$

$$F = \sqrt{3.57^2 + 3.57^2} = 5.1N$$

Electric Field

The concept of electric field can be derived from Coulomb's law.

You take the point of view that the force between two charges is not caused by an instantaneous direct interaction. Rather, the first charge creates a vector field in all of space that we call electric field. Then, the interaction between this electric field at the position of the second charge is the one that generates the electric force. With this understanding, if we have a charge **q** in an electric field **E**, the force on that charge will be:

$$\vec{F} = q\vec{E}$$

That is, we multiply the electric field, which is a vector, times the charge, which is a scalar. The force will be in the same direction as the field if the charge is positive or opposite if it is negative.

Since we measure the force in newton and the charge in coulomb, the units of electric field are N/C. When you learn about electric potential you also learn the definition of the unit volt (represented by V), which gives us the alternative units for the electric field of volt per meter V/m.

Using Coulomb's equation, consider the force on charge q_2 due to the field generated by charge q_1, that is:

$F = k\frac{q_1 q_2}{r^2} = q_1 E$ hence, we get $E = k\frac{q_1}{r^2}$

In general, a charge q generates a field E at a point a distance r away given by the equation:

$$E = k\frac{q}{r^2}$$

Notice that if the charge is positive the vector will be away from the charge as if it were coming from it. If the charge is negative, the field will be directed towards the charge as if sinking into it.

If we need to calculate the field due to several point charges, we add the vectors generated by each charge. This is allowed because the fields can be superimposed.

Continuous Distributions of Charge

There are cases where the electric charges are distributed over a line, a surface, or a volume. We should be able to handle those cases as well.

a) Divide the continuous charge into small differentials. For example, a line can be divided into segments.

b) Write the differential of field as a function of the differential of charge being careful to write the vectors as components. Here, each differential can be considered like a point charge.

c) To find the total electric field you have to integrate each component adding the contributions from all the charges.