Physics II

Electric Potential

The electric potential is a scalar quantity. To specify it at one point all you need is a number with units. It is not necessary information about direction as in the case of the electric field.

Definition: The electric potential V at point $\vec{r}$ is the work that we need to do against the electric force to bring a test charge q_{test} from infinity to the point $\vec{r}$, divided by the value of the test charge.

In equations we can write:

$$V(r) = \frac{W(\text{against } F_{electric})_{\infty \to r}}{q_{test}}$$

Some considerations:

a) In this definition we are taking the potential zero at infinity. That is the conventional reference.

b) We are also assuming that the test charge is so small that it will not perturb the charges that create the potential in the first place.

c) Since the unit for energy is the (J) and the unit of charge is the coulomb (C), the unit of electric potential is J/C which is given the name volt (V).

d) The idea of taking a charge and move it from point to another is only hypothetical. In practice, electric potential, or rather the difference in electric potential, is measured with a voltmeter, which uses the effect of the potential on a resistance to do the measurement.

e) The path followed by the test charge is not important. Any path will give the same result. Only the initial and final points are significant.

f) One way to interpret the electric potential is that if a charge moves from a potential V_1 to a potential V_2, the electric force will do work on the charge as follows:

$$W_{electric, 1 \to 2} = q\left(V_1 - V_2\right)$$

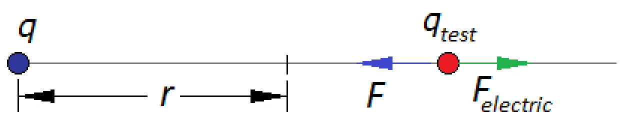
But if the work is done against the electric force:

$$W_{\text{against electric force}, 1 \to 2} = q\left(V_2 - V_1\right)$$

Potential produced by a point charge

Since we know the Coulomb force for point charges, we can calculate the potential as well. Let us see how this is done.

Consider a charge *q* at the origin and we want the electric field at a distance r, as shown below.



The electric force on the test charge q_{test} is:

$$F_{electric} = k\frac{qq_{test}}{x^2}$$

Where x is the distance between the charges.
The green arrow shows the direction of the electric force is the two charges have the same sign. The force we need to apply is the opposite of the electric force, shown as the blue arrow in the figure.

$$F = -k\frac{qq_{test}}{x^2}$$

Since the force is variable, we cannot find the work by simply multiplying force by distance. Instead, we divide the distance in differentials *dx* and integrate from ∞ to r.

$$W = \int_{\infty}^{r} -k\frac{qq_{test}}{x^2}\,dx$$

Integrating we get $W = k\frac{qq_{test}}{x}\Big|_{\infty}^{r} = k\frac{qq_{test}}{r}$

Finally dividing by the test charge, we find the electric potential produced by a point charge:

$$V = k\frac{q}{r}$$

We notice that it has the same sign as the charge and decays with the inverse of the distance.