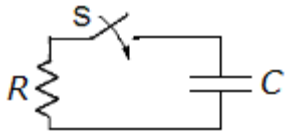
Physics II

RC Circuits

These electric circuits contain a combination of resistors and capacitors.
Consider the following circuit.



Let's say the capacitor initially has a charge q and hence an initial voltage $V_o = \frac{q}{C}$, but at time t=0 the switch S is closed.

Initially, the current will be $i = \frac{V_o}{R}$, but this current will discharge the capacitor, which in turn will reduce the voltage reducing the current as well.

While the capacitor is discharging, the circuit equation will be:

$$\frac{dV}{dt} = -\frac{V}{RC}$$

The minus sign is because the current is due to the capacitor discharge $i = -dq/dt$ and the charge in the capacitor is $q = CV$.

This is an ordinary first order differential equation, whose solution is an exponential function:

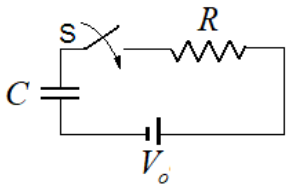
$$V = V_o \exp(-t/\tau)$$

Where $\tau = RC$ is the time constant of the circuit. Notice that 1 farad times 1 ohm is 1 second.

Similarly, the current will decrease exponentially:

$$i = \frac{V_o}{R} \exp(-t/\tau)$$

If the initial conditions or final conditions are different, the solution is similar with the same time constant.
For example, let's examine this circuit:



In this case the capacitor is initially discharged and at t=0 we close the switch S. The initial current will be $i = \frac{V_o}{R}$, but as the capacitor charges, the voltage on the resistor will drop as well as the current.

This time the solution is:

$$V = V_o\left[1 - \exp(t/\tau)\right]$$

Here the voltage in the capacitor will get closer and closer to the source voltage exponentially.