# Physics II

## Capacitors and Dielectrics

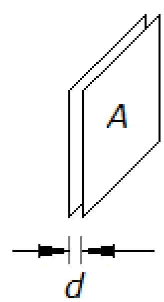
**Electric capacity**

Consider two electrodes with equal and opposite electric charges. Their electric capacity or capacitance is the charge stored divided by the potential difference between them.

$$C = \frac{Q}{V}$$

Since charge is measured in coulomb and potential in volt, the units for capacitance are coulomb/volt also given the name farad (F).

**Case 1, Parallel plates capacitor.-** If two plates with area *A* are set face to face with a small separation *d* between them, we have a parallel plates capacitor.



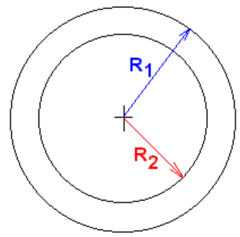

Consider that the density of charge in one surface is $\sigma$ and $-\sigma$ in the other. The electric field between the plates will be $E = \frac{\sigma}{\varepsilon_\circ}$, a uniform value, the potential difference between the plates will be $V = \frac{\sigma}{\varepsilon_\circ} d$, and the charge on the positive plate is $Q = A\sigma$. We apply the definition of capacitance to these equations obtaining:

$$C = \frac{Q}{V} = \frac{A\sigma}{\frac{\sigma}{\varepsilon_\circ} d}, \text{ so}$$

$$C = \varepsilon_\circ \frac{A}{d}$$

Where $\varepsilon_o = 8.85 \times 10^{-12}\, F/m$

**Case 2, Spherical Capacitor.-** Consider two conducting concentric spheres with opposite charges Q in the internal sphere and –Q in the external one.




The difference in potential between the spheres is:

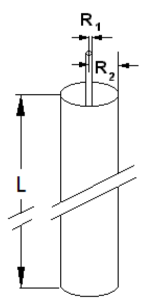
$$V = \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R_2} - \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R_1}$$

And the capacitance:

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R_2} - \frac{1}{4\pi\varepsilon_\circ}\frac{Q}{R_1}}$$

$$C = 4\pi\varepsilon_\circ\frac{R_1R_2}{R_1 - R_2}$$

**Case 3, Cylindrical Capacitor.-** Consider two cylindrical conductors with radii $R_1$ and $R_2$ and length L.




If the internal conductor has a charge $Q = L\lambda$ and the external one $-Q$ then the electric field between them is

$$E = \frac{\lambda}{2\pi\varepsilon_\circ r} = \frac{Q}{2\pi\varepsilon_\circ rL}$$

Where $r$ is the distance to the axis.

To calculate the difference in potential we integrate the electric field between the two radii.

$$V = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_\circ rL} dr = \frac{Q}{2\pi\varepsilon_\circ L} \ln\left(\frac{R_2}{R_1}\right)$$

The capacitance is:

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi\varepsilon_\circ L}\ln\left(\frac{R_2}{R_1}\right)}$$

$$C = \frac{2\pi\varepsilon_\circ L}{\ln\left(\frac{R_2}{R_1}\right)}$$

**Dielectric Materials**

Dielectric materials are insulators that are polarized when they are in the presence of an electric field, different than conductors where electric currents appear. Polarizing a material means to create a distortion or asymmetry in its internal charges to generate an internal field opposite to the external one. In consequence, the electric field inside a dielectric is reduced with respect to the vacuum by a factor $1/K$, where K is called the dielectric constant.

If a capacitor is filled with a dielectric the voltage will be reduced, so the capacitance will be larger. For example, for a parallel plates capacitor the capacitance will be:

$$\mathrm{C} = K\varepsilon_\circ \frac{\mathrm{A}}{\mathrm{d}}$$

**Energy stored in a capacitor**

Consider a capacitor to which we add charges $dq$ starting from an initial neutral state. The accumulated charges will produce a voltage $\mathrm{V} = \frac{\mathrm{q}}{\mathrm{C}}$ and each charge will add an energy $\mathrm{dW} = \frac{\mathrm{q}}{\mathrm{C}} dq$ to the capacitor.

The total energy can be calculated by integrating:

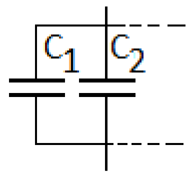
$$\text{Energy} = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Alternatively we can use the definition of capacitance to write this energy in terms of voltage

$$\text{Energy} = \frac{1}{2}\mathrm{CV}^2$$

**Capacitors in parallel**

Consider two or more capacitors in parallel.




We notice that the total charge will be the sum of the individual charges:

$$\mathrm{Q} = Q_1 + Q_2 + \ldots$$

All the capacitors have the same voltage, so we can write the equation:

$$\mathrm{Q} = C_1 V + C_2 V + \ldots$$

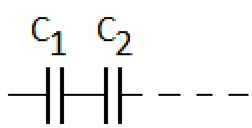
With the definition of capacitance we find:

$$\mathrm{C} = C_1 + C_2 + \ldots$$

The equivalent of capacitors in series is the sum of the individual capacitances.

**Capacitors in series**

Consider capacitors connected in series:




The difference in potential will be the sum of the individual voltages:

$$\mathrm{V} = V_1 + V_2 + \ldots$$

Because they are in series, they will have the same current through, so the same charge as well, then:

$$\mathrm{V} = \frac{Q}{C_1} + \frac{Q}{C_2} + \ldots$$

Using the definition of capacitance:

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2} + \ldots}$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots}$$

The equivalent capacity of capacitors in series us the inverse of the sum of the inverses of the individual capacitances.

Notice that these calculations with capacitors are similar to resistors and inductors, but in reverse.