Physics II

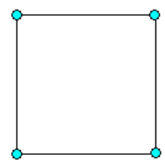
Electric Field and Force

$F = k\dfrac{q_1 q_2}{d^2}$ Coulomb's law, where $k = 9\times10^9\,\mathrm{Nm^2/C^2}$

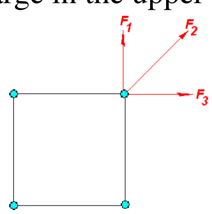
$E = \dfrac{F_{test}}{q_{test}}$ Definition of electric field. This is a vector.

$E = k\dfrac{Q}{d^2}$ Electric field for a point charge. This is a vector.

Problem 1.- A charge of q = 6μC is placed at each corner of a square 0.35m on a side. Determine the magnitude of the force on each charge.



Solution: Consider the forces on the charge in the upper right corner:



$$F_1 = k\frac{q_1 q_2}{r^2} = 9\times10^9\,\frac{(6\times10^{-6})(6\times10^{-6})}{0.35^2} = 2.64$$

$$F_2 = k\frac{q_1 q_2}{r^2} = 9\times10^9\,\frac{(6\times10^{-6})(6\times10^{-6})}{(0.35\sqrt{2})^2} = 1.32$$

$$F_3 = k\frac{q_1 q_2}{r^2} = 9\times10^9\,\frac{(6\times10^{-6})(6\times10^{-6})}{0.35^2} = 2.64$$

To add these vectors, we have to find their components:

$F_1 = (0,2.64)$

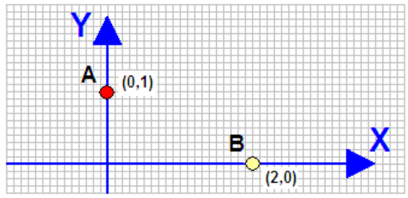
$F_2 = (1.32\cos45^\circ, 1.32\sin45^\circ) = (0.93,0.93)$

$\underline{F_3 = (2.64,0)}$

$F = (3.57,3.57)$

$F = \sqrt{3.57^2 + 3.57^2} = 5.1\mathrm{N}$

Problem 2.- Determine the electric field magnitude at the origin due to the charges located at A and B. Consider the coordinates given in meters and charge $q_A = -2\mu C$ and $q_B = -1\mu C$.



Solution: To find the electric field we can first calculate the magnitudes:

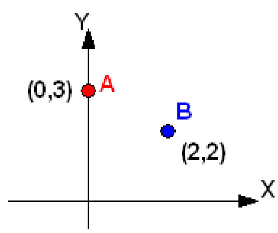
$$E_A = k\frac{q_A}{r_A^2} = 9\times10^9\frac{2\times10^{-6}}{(1)^2} = 18{,}000\text{V/m}$$

$$E_B = k\frac{q_B}{r_B^2} = 9\times10^9\frac{1\times10^{-6}}{(2)^2} = 2{,}250\text{V/m}$$

The magnitude of the resulting vector will be:

$$E = \sqrt{E_A^2 + E_B^2} = \mathbf{18{,}140\ V/m}$$

Problem 3.- Determine the electric ***field*** at the origin of coordinates due to the charges located at A and B. Consider the coordinates given in meters and charges $Q_A = 4\mu C$ and $Q_B = -5\mu C$.
In your answer write the vector as components and find its magnitude.



Solution: To find the electric field we calculate the contribution from each charge and add the vectors:

$$E_A = k\frac{Q_A}{r^2} = 9\times10^9\frac{4\times10^{-6}}{3^2} = 4{,}000\text{ V/m}$$

$$E_B = k\frac{Q_B}{r^2} = 9\times10^9\frac{5\times10^{-6}}{(2\sqrt{2})^2} = 5{,}625\text{ V/m}$$

The vector produced by Q_A at the origin is downward because it is a positive charge, so:

$$E_A = (0, -4000)$$

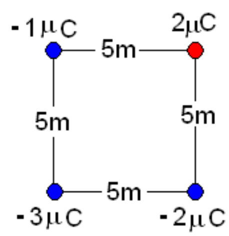
Charge Q_B produces an attractive force on a positive test charge, so the vector will be from the origin towards point B. This direction is 45º above the positive x-axis. The vector is:

$$E_B = (5625\cos 45^\circ, 5625\cos 45^\circ) = (3977, 3977)$$

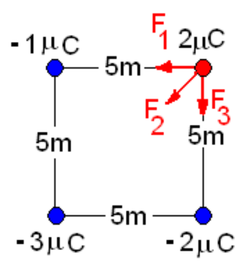
The resultant vector is: $\vec{E} = (3977, -23)$

Which has a magnitude of $|E| = \sqrt{3977^2 + 23^2}$ = **3,978 V/m**

Problem 4.- Calculate the net force on the positive charge on the right upper corner of the square shown in the figure. Give your answer in magnitude of the vector.



Solution: There are three forces acting on the positive charge as shown in the figure below. The magnitudes are calculated using Coulomb's law.



$$F_1 = k\frac{q_1q_2}{d^2} = 9\times10^9\frac{(2\times10^{-6})(1\times10^{-6})}{5^2} = 7.2\times10^{-4}\text{N}$$

$$F_2 = k\frac{q_1q_2}{d^2} = 9\times10^9\frac{(2\times10^{-6})(3\times10^{-6})}{7.07^2} = 1.08\times10^{-3}\text{N}$$

$$F_3 = k\frac{q_1q_2}{d^2} = 9\times10^9\frac{(2\times10^{-6})(2\times10^{-6})}{5^2} = 1.44\times10^{-3}\text{N}$$

To add these vectors, we need to write them as components, so:

$$F_1 = (-7.2\times10^{-4}, 0)$$

$$F_2 = (-1.08\times10^{-3}\cos45^\circ, -1.08\times10^{-3}\sin45^\circ) = (-7.64\times10^{-4}, -7.64\times10^{-4})$$

$$F_3 = (0, -1.44\times10^{-3}\text{N})$$

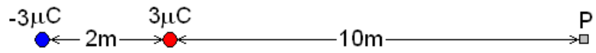
Adding the components, we get:

$$F_1 + F_2 + F_3 = (-14.84\times10^{-4}, -22.04\times10^{-4})$$

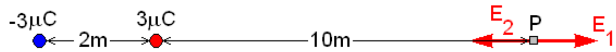
And the magnitude is:

$F = \sqrt{(14.84\times10^{-4})^2 + (22.04\times10^{-4})^2}$ = **2.6×10^{-3}N**

Problem 5.- Calculate the electric field at point "P" in the figure:



Solution: The positive charge produces an electric field pointing to the right, as shown.



With a magnitude equal to:

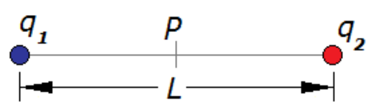
$$E_1 = k\frac{q}{d^2} = 9\times10^9\,\frac{3\times10^{-6}}{10^2} = 270\text{V/m}$$

The negative charge instead produces an electric field pointing to the left, with a magnitude equal to:

$$E_2 = k\frac{q}{d^2} = 9\times10^9\,\frac{3\times10^{-6}}{12^2} = 187.5\text{V/m}$$

Adding the vectors, we get $E =$ **82.5 V/m** pointing to the right.

Problem 6.- Determine the magnitude and direction of the electric field at a point *P* midway between charges q_1=-16μC and q_2=+11.2μC that are a distance *L*=10cm apart.



Solution: Each charge will produce an electric field at point P. The vectors will be:

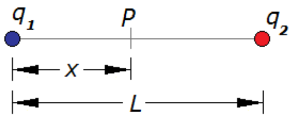
$$E_1 = k\frac{q_1}{d^2} = 9\times10^9\,\frac{16\times10^{-6}}{0.05^2} = 57.6\times10^6\,\text{V/m}$$ pointing to the left.

$$E_2 = k\frac{q_2}{d^2} = 9\times10^9\,\frac{11.2\times10^{-6}}{0.05^2} = 40.3\times10^6\,\text{V/m}$$ pointing to the left as well.

Since the two vectors point to the left the sum will be:

$40.3\times10^6\,\text{V/m} + 57.6\times10^6\,\text{V/m} = 97.9\times10^6\,\text{V/m}$ to the left.

Problem 7.- Determine the position of a point P between a charge q_1=-4μC and another q_2=+1μC that are at a distance L=1m apart where the electric field has the minimum value.



Solution: The electric field (total value) is: $E = \frac{k(4\mu C)}{x^2} + \frac{k(1\mu C)}{(L-x)^2}$

Taking the first derivative, we get: $\frac{dE}{dx} = -2\frac{k(4\mu C)}{x^3} + 2\frac{k(1\mu C)}{(L-x)^3}$

To get the minimum value, the derivative must be equal to zero, so:

$$\frac{dE}{dx} = -2\frac{k(4\mu C)}{x^3} + 2\frac{k(1\mu C)}{(L-x)^3} = 0 \rightarrow -\frac{4}{x^3} + \frac{1}{(L-x)^3} = 0$$

Solving for x:

$$\rightarrow -\frac{4}{x^3} + \frac{1}{(L-x)^3} = 0 \rightarrow x = \frac{\sqrt[3]{4}L}{1+\sqrt[3]{4}}$$

$$\rightarrow x = 0.61L$$

Problem 8.- Calculate the electric force between an alpha particle and an aluminum nucleus separated by 0.529×10^{-10} m
Charge of an alpha particle 3.2×10^{-19} C
Charge of an aluminum nucleus 20.8×10^{-19} C

Solution: The equation needed is Coulomb's law. $\mathrm{F} = \mathrm{k}\frac{\mathrm{q_1 q_2}}{\mathrm{r}^2}$, with the values of the problem:

$$\mathrm{F} = 9\times10^9 \frac{(3.2\times10^{-19})(20.8\times10^{-19})}{(0.5\times10^{-10})^2} = \mathbf{2.4\times10^{-6}\ N}$$

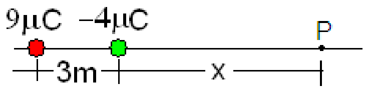
Problem 9.- Find the acceleration experienced by an electron in an electric field of 250 V/m
Charge of the electron $q_e = -1.6\times10^{-19}$ C and mass $m_e = 9.1\times10^{-31}$kg

Solution: The definition of electric field is force per charge: $E = \frac{F}{q}$, so $F = qE$.

We also know that $F = ma$ or $a = \frac{F}{m} = \frac{qE}{m}$

With the values given: $a = \frac{qE}{m} = \frac{(1.6\times10^{-19}\,C)(250N/C)}{9.1\times10^{-31}\,kg} =$ $\mathbf{4.4\times10^{13}\ m/s^2}$

Problem 10.- Given the two charges shown in the figure, at what position "x" is the electric field zero?



Solution: The electric field due to the positive charge is $E_p = k\frac{9\mu C}{(3+x)^2}$ and the one due to the negative charge is $E_n = k\frac{4\mu C}{x^2}$, but in the opposite direction, so the net field will be zero if the two vectors have the same magnitude:

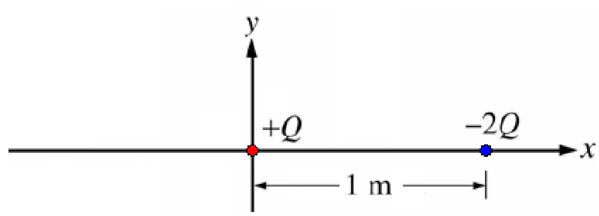
$$k\frac{9\mu C}{(3+x)^2} = k\frac{4\mu C}{x^2}$$

Simplifying: $\frac{9}{(3+x)^2} = \frac{4}{x^2}$

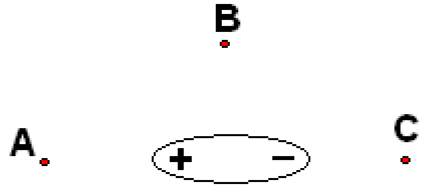
Solving for x:

$$\frac{3}{3+x} = \frac{2}{x} \rightarrow 3x = 6+2x \rightarrow x = \mathbf{6m}$$

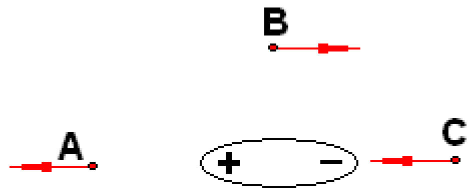
Problem 10a.- A charge of +*Q* coulombs is placed at the origin and a charge of -2*Q* coulombs is placed at x=+1 meter as shown in the figure. At what point on the x-axis will a test charge of +q coulombs experience zero net force?



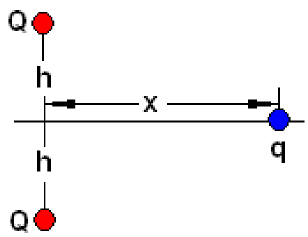
Problem 11.- Draw arrows to indicate the direction of the electric field at points **A**, **B** and **C** due to the dipole shown in the figure.



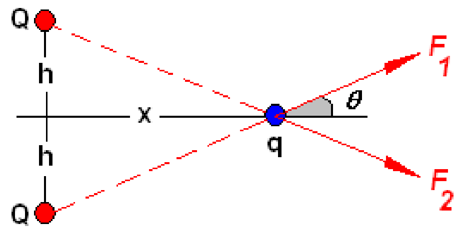
Solution:



Problem 12.- In the following arrangement find an expression for the net force on the charge “q” due to the other two charges “Q” and find the value of “x” that makes the force maximum.



Solution:



There are two forces acting on the charge, F_1 and F_2:

$$\vec{F}_1 = \frac{kQq}{h^2 + x^2}(\cos\theta, \sin\theta)$$

$$\vec{F}_2 = \frac{kQq}{h^2 + x^2}(\cos\theta, -\sin\theta)$$

The sum of the forces is: $F_1 + F_2 = \dfrac{kQq}{h^2 + x^2}(2\cos\theta, 0)$

Writing the cosine of θ in terms of x and h:

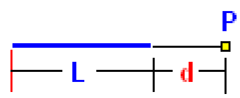
$$F_1 + F_2 = \frac{kQq}{h^2 + x^2}\left(\frac{2x}{\sqrt{h^2 + x^2}}, 0\right)$$

We find the maximum force in the usual way, taking the derivative and finding where it is zero.

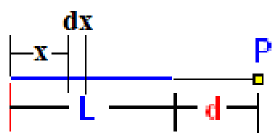
$$\frac{d}{dx}\frac{2kQqx}{\left(h^2+x^2\right)^{3/2}}=\frac{2kQq\left(h^2+x^2\right)^{3/2}-2kQqx3/2\left(h^2+x^2\right)^{1/2}(2x)}{\left(h^2+x^2\right)^3}=0$$

$$\rightarrow h^2+x^2-3x^2=0\rightarrow x=\frac{h}{\sqrt{2}}$$

Problem 13.- We want to find the electric field at point "P" due to a wire with constant linear density of charge λ=5μC/m and length L=1m located a distance d=0.5m from point P as shown in the figure.



Solution: Consider a differential of wire as shown below.



It will have a charge $dq=\lambda dx$ and it is located at a distance L+d-x from point P, so it will contribute an electric field equal to:

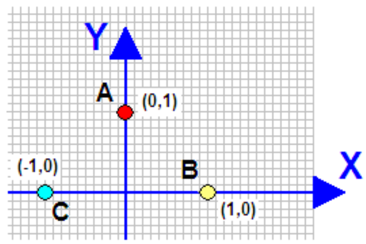
$$dE=\frac{k\lambda dx}{(L+d-x)^2},$$

Integrating to find E

$$E=\int_0^L\frac{k\lambda dx}{(L+d-x)^2}$$

$$E=\left.\frac{k\lambda}{L+d-x}\right|_0^L=\frac{k\lambda}{d}-\frac{k\lambda}{L+d}=\frac{k\lambda L}{d(L+d)}=\frac{(9\times10^9)(5\times10^{-6})(1)}{0.5(1+0.5)}=60,000V/m$$

Problem 14.- Determine the electric field (magnitude and direction) at the origin due to the charges located at A, B and C. Consider the coordinates given in meters and charge of q_A=-2μC, q_B=-1μC and q_C=1μC.



Solution: First, we find the magnitudes of the three vectors using the equation for the electric field of a point charge:

$$E_A = k\frac{q_A}{r^2} = 9\times10^9\frac{2\times10^{-6}}{(1)^2} = 18{,}000\text{N/C}$$

$$E_B = k\frac{q_B}{r^2} = 9\times10^9\frac{1\times10^{-6}}{(1)^2} = 9{,}000\text{N/C}$$

$$E_C = k\frac{q_C}{r^2} = 9\times10^9\frac{\text{Nm}^2}{\text{C}^2}\frac{1\times10^{-6}\text{C}}{(1\text{m})^2} = 9{,}000\text{N/C}$$

Now we consider the directions of these vectors:

E_A: Since q_A is negative, the electric field vector will point towards the charge, so it will be a vertically upward vector.
(0, 18000N/C)

E_B: Since q_B is also negative, the electric field vector will point towards the charge, so it will be a horizontal vector pointing to the right.
(9000 N/C, 0)

E_C: q_C is positive, so the electric field vector will point away from the charge. It will be a horizontal vector pointing to the right.
(9000 N/C, 0)

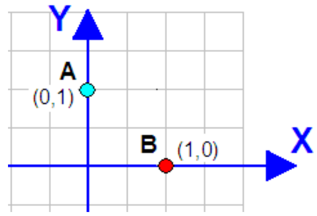
We can add these three vectors:

$E_A + E_B + E_C$ = (18000 N/C, 18000 N/C):

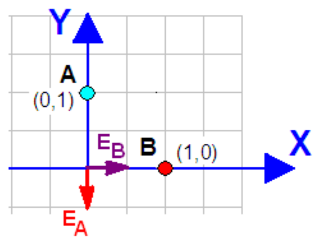
$$E = \sqrt{\left(\sum Ex\right)^2 + \left(\sum Ey\right)^2} = \sqrt{18{,}000^2 + 18{,}000^2} = \mathbf{25{,}500\ N/C}$$

The direction of this vector will make **45º** with the horizontal.

Problem 15.- Determine the magnitude of the electric field at the origin due to the two charges "A" and "B". Consider the positions given in meters and charge of A=30μC and B=-30μC.



Solution: A sketch of the electric field vectors:



The magnitude of each electric field is:

$$E_A = E_B = k\frac{q_a}{r^2} = 9\times10^9 \frac{(30\times10^{-6})}{(1.0)^2} = 2.7\times10^5 \frac{V}{m}$$

Regarding the direction of the vectors:

Notice that q_A produces an electric field pointing downwards, because the force on a positive test charge at the origin would be repulsive. Instead q_B, which is negative, would produce an attractive force on a positive test charge at the origin, so its electric field is a vector that points to the right.

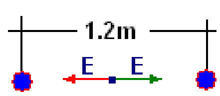
$$E = (2.7\times10^5 \text{ V/m}, -2.7\times10^5 \text{ V/m})$$

The magnitude of the vector is:

$$F = \sqrt{(2.7\times10^5 \text{ V/m})^2 + (2.7\times10^5 \text{ V/m})^2} = \mathbf{3.8\times10^5 \text{ V/m}}$$

Problem 16.- An 18μC charge is placed 1.2 m from an identical 18μC charge. Calculate the electric field and the electric potential at the point midway between them.

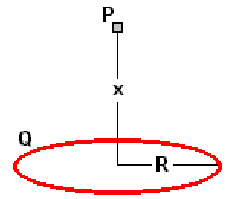
Solution: The electric field produced by each charge will be the same in magnitude, but one towards the left and one towards the right, so when you add them together you get **zero**.



Electric potential is a scalar quantity, so we need to add the potential produced by each charge. The distance that goes in the formula "r" is the distance from the point midway between the charges and each charge, so it is 0.6 m (not 1.2 m) and the potential is:

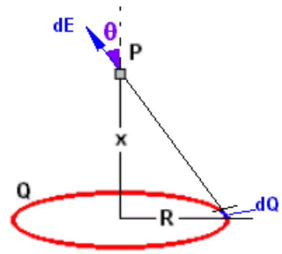
$$\mathrm{V} = 9\times10^9 \frac{18\times10^{-6}}{0.6} + 9\times10^9 \frac{18\times10^{-6}}{0.6} = \mathbf{540{,}000\ V}$$

Problem 17.- The ring shown in the figure has a uniform charge Q and radius R. Determine the electric ***field*** at point P, which is located on the axis of the ring a distance "x" from its center. In the same charge distribution, find the electric ***potential*** at point P.



Solution: To calculate the electric field, notice that we can divide the ring into differentials of charge. Each one contributing an electric field equal to:

$$\mathrm{dE} = k\frac{dQ}{r^2} = k\frac{dQ}{x^2+R^2}$$



Only the component of the vector in the vertical direction will contribute to the integral. The other components will give zero due to the symmetry of the problem, so:

$$\mathrm{dE_x} = k\frac{dQ}{x^2+R^2}\cos\theta = k\frac{dQ}{x^2+R^2}\frac{x}{\sqrt{x^2+R^2}}$$

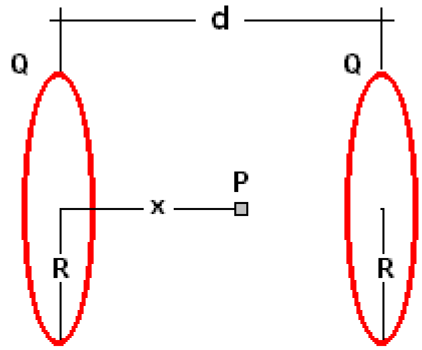
After integrating, we get

$$\mathrm{E} = k\frac{Qx}{\left(x^2+R^2\right)^{3/2}}$$

If we want the electric ***potential*** at point P, we do not need to be concerned with vectors, we just add (integrate) all the contributions:

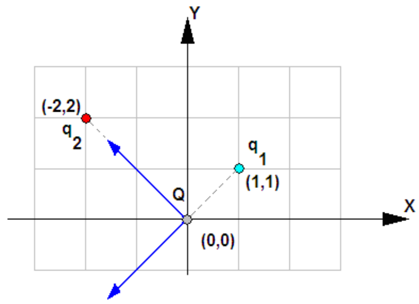
$$\mathrm{dV} = k\frac{dQ}{r} = k\frac{dQ}{\sqrt{x^2+R^2}} \rightarrow \mathrm{V} = \frac{kQ}{\sqrt{x^2+R^2}}$$

Problem 17a.- Each of the rings shown in the figure has uniform charge Q and radius R. Determine the electric ***field*** at point P, which is located on the axis of the rings a distance "x" from the center of the left ring.



Problem 18.- Find the net force on a charge Q=8.5μC located at the origin of coordinates (0,0) due to a charge q_1=1.5μC located at (1.0m, 1.0m) and another charge q_2=-2.0μC located at (-2.0m, 2.0m).

Solution: Notice that the two forces form 90º with respect to each other:



Since Q and q_1 have the same sign, the force between these charges will be repulsive. On the other hand, q_2 is negative, so it will attract Q.
Coulomb's Law gives the magnitudes of the forces.

$$F_{Qq_1} = k\frac{Qq_1}{r_{Qq_1}^2} = 9\times10^9\frac{(8.5\times10^{-6})(1.5\times10^{-6})}{1^2+1^2} = 5.74\times10^{-2}\,\text{N}$$

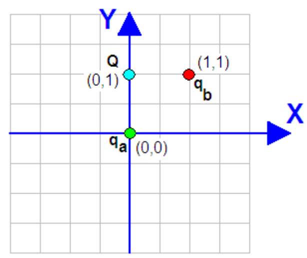
$$F_{Qq_2} = k\frac{Qq_2}{r_{Qq_2}^2} = 9\times10^9\frac{(8.5\times10^{-6})(2\times10^{-6})}{2^2+2^2} = 1.91\times10^{-2}\,\text{N}$$

Since the two vectors are orthogonal to each other, we can use Pythagoras' theorem to find the magnitude of the sum:

$$F = \sqrt{F_{Qq1}^2 + F_{Qq2}^2} = \sqrt{(5.74\times10^{-2})^2 + (1.91\times10^{-2})^2} = \mathbf{0.0605\,N}$$

Problem 19.-Find the magnitude of the net force on a charge Q=1.5μC located at position (0, 1.0 m) due to a charge $q_a = 2.5\mu C$ located at the origin (0,0) and another charge $q_b = -5\mu C$ located at (1.0m,1.0m).

Solution: A sketch of the problem:



Coulomb's Law gives the magnitude of each force.

$$F_{Qqa} = k\frac{Qq_a}{r^2_{Qqa}} = 9\times10^9\frac{(1.5\times10^{-6})(2.5\times10^{-6})}{(1.0)^2} = 0.0338\text{ N}$$

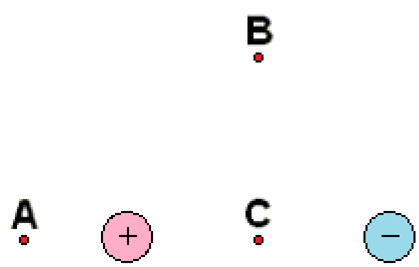
$$F_{Qqb} = k\frac{Qq_b}{r^2_{Qqb}} = 9\times10^9\frac{(1.5\times10^{-6})(5.0\times10^{-6})}{(1.0)^2} = 0.0675\text{ N}$$

Regarding the direction of the vectors, notice that Q and q_a have the same sign, so the force is repulsive. It will be in the positive "Y" direction. On the other hand, q_b is negative, so the force on Q is attractive and will be in the positive "X" direction. The vector force will be then:

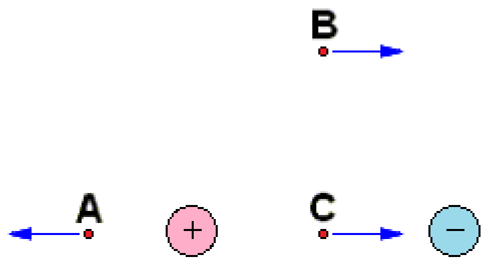
$$\vec{F} = (0.0338\text{ N}, 0.0675\text{ N})$$

The magnitude of the vector is: $F = \sqrt{(0.0338\text{ N})^2 + (0.0675\text{ N})^2}$ = **0.0755 N**

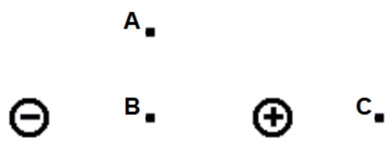
Problem 20.- Indicate the direction of the electric field at points A, B and C due to the two charges shown in the figure as "+" and "-". Consider the charges identical in magnitude, but one positive and the other negative, as shown.



Solution: The direction of the electric field vectors at points A, B and C are shown:



Problem 20a.- Indicate the direction of the electric field at points A, B and C due to the two charges shown in the figure. Consider the charges identical in magnitude, but positive and negative as indicated.



Solution: The direction of the electric field vectors at points A, B and C are:

Problem 21.- A sphere of 1.0 m radius is uniformly charged over its entire volume with total charge Q = 1.5×10^{-6}C. Find the electric field inside the sphere at a distance of 0.5m from its center.

Solution: Since the charge and radius of the sphere are given, we can calculate the density of charge:

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

Now, applying Gauss's law at a point 0.5m form the center:

$$E_{\perp} Area = \frac{q_{enclosed}}{\varepsilon_o} \rightarrow E = \frac{q_{enclosed}}{Area(\varepsilon_o)} = \frac{\rho(Volume\ Enclosed)}{Area(\varepsilon_o)}$$

Plugging in the density and volume enclosed:

$$E = \frac{\frac{Q}{\frac{4}{3}\pi R^3}\left(\frac{4}{3}\pi(0.5m)^3\right)}{4\pi(0.5m)^2(\varepsilon_o)} = \frac{\frac{Q}{R^3}(0.5m)^3}{4\pi(0.5m)^2(\varepsilon_o)}$$

With the values of R, Q and ε_o we get:

$$E = 9\times10^9 \frac{Nm^2}{C^2}\frac{\frac{1.5\times10^{-6}C}{(1m)^3}(0.5m)^3}{(0.5m)^2} = \textbf{6,750 N/C}$$

Problem 22.- If a positively charged particle enters a region of uniform electric field which is perpendicular to the particle's initial velocity, will the kinetic energy of the particle increase, decrease or stay the same? Why?

Solution: The electric force will produce acceleration in a direction that is perpendicular to the initial velocity of the particle. As it will pick up velocity in that direction, its kinetic energy will increase.

Problem 23.- Determine the electric ***field*** at the origin of coordinates due to the charges located at A, B and C. Consider the coordinates given in meters and charges Q_A=4μC, Q_B=4μC and Q_B=5μC.
In your answer, write the vector as components.