

Physics II

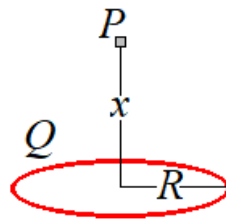
Electric Field and Force, 3D

$\vec{E} = \frac{\vec{F}_{test}}{q_{test}}$ Definition of electric field. This is a vector.

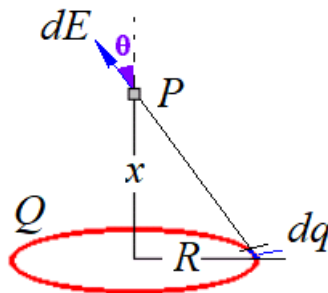
$F = k \frac{q_1 q_2}{d^2}$ Coulomb's law, where $k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

$E = k \frac{Q}{d^2}$ Electric field produced by a point charge Q at a distance d .

Problem 1.- The ring shown in the figure has a uniformly distributed charge Q and radius R . Determine the electric field at point P , which is located on the axis of the ring a distance x from its center. In the same charge distribution, find the electric potential at point P .



Solution: To calculate the electric field, notice that we can divide the ring into differentials of charge.



Each differential of charge creates an electric field equal to:

$$dE = k \frac{dQ}{r^2} = k \frac{dQ}{x^2 + R^2}$$

Only the component of the vector in the vertical direction will contribute to the integral. The other components will give zero due to the symmetry of the problem, so:

$$dE_x = k \frac{dQ}{x^2 + R^2} \cos \theta = k \frac{dQ}{x^2 + R^2} \frac{x}{\sqrt{x^2 + R^2}}$$

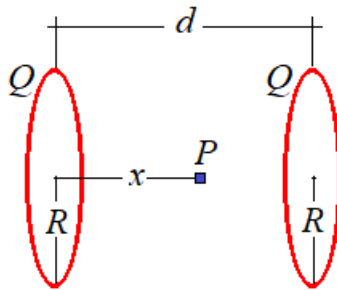
After integrating, we get

$$E = k \frac{Qx}{(x^2 + R^2)^{3/2}}$$

If we want the electric potential at point P , we do not need to be concerned with vectors, we just add (integrate) all the contributions:

$$dV = k \frac{dQ}{r} = k \frac{dQ}{\sqrt{x^2 + R^2}} \rightarrow V = \frac{kQ}{\sqrt{x^2 + R^2}}$$

Problem 1a.- Each of the rings shown in the figure has uniformly distributed charge Q and radius R . Determine the electric field at point P , which is located on the axis of the rings a distance x from the center of the left ring.



Solution: To calculate the electric field, we can add the vectors produced by the two rings:

$$E = k \frac{Qx}{(x^2 + R^2)^{3/2}} - k \frac{Q(d-x)}{((d-x)^2 + R^2)^{3/2}}$$

Problem 2.- If a positively charged particle enters a region of uniform electric field which is perpendicular to the particle's initial velocity, will the kinetic energy of the particle increase, decrease or stay the same? Why?

Solution: The electric force will produce acceleration in a direction that is perpendicular to the initial velocity of the particle. As it will pick up velocity in that direction, its kinetic energy will increase.

Problem 3.- You have a cylinder with total charge $Q = 1\text{nC}$ uniformly distributed over all its volume, radius $R = 0.1\text{m}$, height $h = 4\text{m}$ and you want to find the electric field at a point 10m below its base.

Solution: Since the cylinder is so narrow, it can be approximated by a wire with linear charge density:

$$\lambda = \frac{1\text{nC}}{4\text{m}} = 0.25\text{nC/m}$$

We divide its height in small disks and calculate the electric field contributed by this differential of charge.

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dz}{(10-z)^2}$$

Integrating

$$E = \frac{\lambda}{4\pi\epsilon_0} \int_0^h \frac{dz}{(10-z)^2} = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{10-h} - \frac{1}{10} \right)$$

And with the values given $E = \mathbf{0.15 \text{ V/m}}$