Physics II

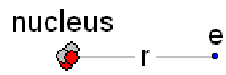
Electric Potential

$$V = \frac{\text{Work}_{\text{test}}}{q_{\text{test}}}$$ Definition of electric potential.

$$V = k\frac{Q}{d}$$ Electric potential for a point charge.

Problem 1.- Calculate the electric potential energy of an electron in a He^{+} ion. Consider the distance between the nucleus and the electron to be $r = 0.264\times10^{-10}$ m and

Charge of the nucleus	3.2×10^{-19} C
Charge of the electron	-1.6×10^{-19} C

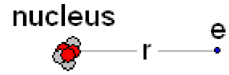


Solution: The value in joules is

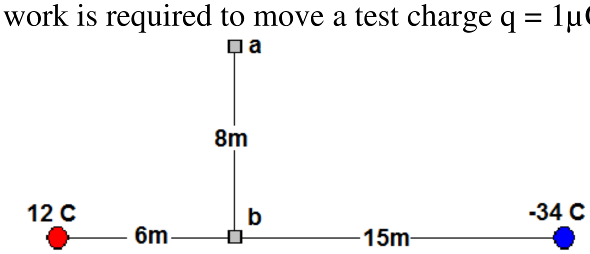
$$Energy = k\frac{q_1 q_2}{d} = 9\times10^{9}\frac{(3.2\times10^{-19})(-1.6\times10^{-19})}{0.264\times10^{-10}} = \mathbf{-\ 1.764\times10^{-17}\ J}$$

Converted to eV it is **-109 eV.**

Problem 1a.- Calculate the electric potential energy of an electron in a Li^{++} ion. Consider the distance between the nucleus and the electron to be $r = 0.176\times10^{-10}$ m and:
The charge of the nucleus is 4.8×10^{-19} C.
The charge of the electron is -1.6×10^{-19} C.



Problem 2.- How much work is required to move a test charge q = 1μC from point a to point b?



Solution: To solve this problem first we calculate the potentials at *a* and *b*:

$$V_a = k\frac{12}{10} + k\frac{(-34)}{17} = -7.2\times10^{9}\,volt$$

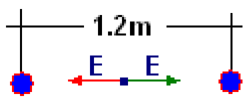
$$V_b = k\frac{12}{6} + k\frac{(-34)}{15} = -2.4\times10^9\,volt$$

To find the work we calculate the difference in potential and multiply by the test charge:

$W = (V_b - V_a)q_{test} = (-2.4\times10^9 + 7.2\times10^9)(1\times10^{-6}) =$ **4,800 J**

Problem 3.- An 18μC charge is placed 1.2 m from an identical 18μC charge. Calculate the electric field and the electric potential at the point midway between them.

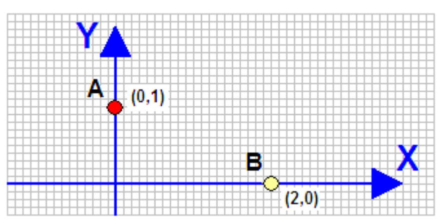
Solution: The electric field produced by each charge will be the same in magnitude, but one towards the left and one towards the right, so when you add them together you get **zero**.



Electric potential is a scalar quantity, so we need to add the potential produced by each charge. The distance that goes in the formula "r" is the distance from the point midway between the charges and each charge, so it is 0.6 m (not 1.2 m) and the potential is:

$\mathrm{V} = 9\times10^9 \frac{18\times10^{-6}}{0.6} + 9\times10^9 \frac{18\times10^{-6}}{0.6} =$ **540,000 V**

Problem 4.- Determine the electric potential at the origin due to the charges located at A and B. Consider the coordinates given in meters and charge $q_A = -2\mu C$ and $q_B = -1\mu C$.



Solution: Each charge will contribute a potential and we need to add those two values.

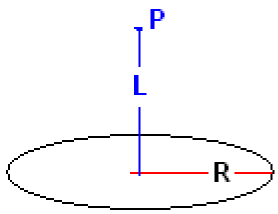
$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} = 9\times10^9 \frac{(-2\times10^{-6})}{1} + 9\times10^9 \frac{(-1\times10^{-6})}{2} =$ **-22,500 V**

Problem 5.- An alpha particle, which is a helium nucleus with $mass = 6.64\times10^{-27}\,kg$, is emitted in a radioactive decay with kinetic energy $KE = 7.8\times10^6\,eV$. Calculate its speed.

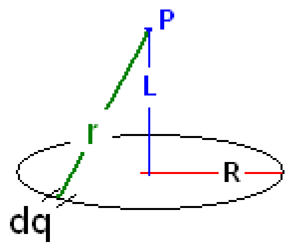
Solution: The kinetic energy in the problem is given in terms of eV, which is the charge of a proton times one volt. For speeds that are not relativistic, we can use the kinetic equation $KE = \frac{1}{2}mv^2$, and then the speed is:

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\times 7.8\times 10^6 (1.6\times 10^{-19})}{6.64\times 10^{-27}}}$$ **= 1.94×10^7 m/s**

Problem 6.- Calculate the electric potential due to a uniform ring of radius R=1.2m that has a total charge of 32μC at a point *P* on the symmetry axis and a distance L=1.6m from the center of the ring.



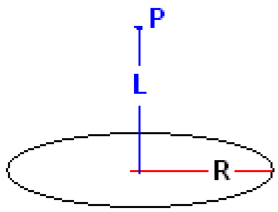
Solution: Notice that the distance from any differential of charge, dq, to P is always the same, namely: $\mathrm{r} = \sqrt{\mathrm{R}^2 + \mathrm{L}^2} = \sqrt{(1.2\mathrm{m})^2 + (1.6\mathrm{m})^2} = 2\mathrm{m}$



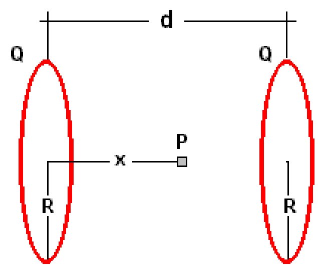
Therefore, the electric potential is

$$\mathrm{V} = \int \frac{\mathrm{kdq}}{\mathrm{r}} = \frac{\mathrm{kq}}{\mathrm{r}} = \frac{9\times 10^9 (32\times 10^{-6})}{2}$$ = **144,000 volts**

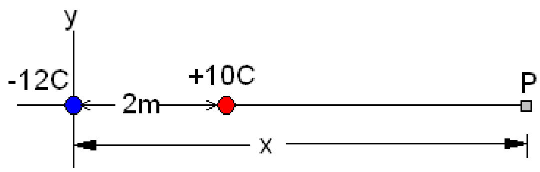
Problem 6a.- Calculate the electric potential due to a disk of radius R=1.2m that has a total charge of 32μC, uniformly distributed on its surface, at a point P on the symmetry axis at a distance L=1.6m from the center of the disk.



Problem 6b.- Each of the rings shown in the figure has uniform charge Q and radius R. Determine the electric potential at P, which is located on the axis of the rings a distance x from the center of the left ring.



Problem 7.- Find the point *P* on the x-axis on the right side of the positive charge where the electric potential is zero:



Solution: The potential produced by the positive charge at P is $k\dfrac{10}{x-2}$ and the one produced by the negative charge is $k\dfrac{(-12)}{x}$, the total potential is zero, so:

$$k\frac{10}{x-2}+k\frac{(-12)}{x}=0 \to x=\mathbf{12m}$$

Problem 8.- Calculate the potential energy of an alpha particle separated a distance 2.5×10^{-10} m from an aluminum nucleus. The charge of an alpha particle is 3.2×10^{-19} C and the one of an aluminum nucleus is 20.8×10^{-19} C.

Solution: We first calculate the electric potential due to the nucleus at the indicated position and then multiply by the charge of the alpha particle.

$$\mathrm{V}=\mathrm{k}\frac{\mathrm{q}}{\mathrm{r}}=9\times10^{9}\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\,\frac{3.2\times10^{-19}\,\mathrm{C}}{2.5\times10^{-10}\,\mathrm{m}}=11.52\mathrm{V}$$

E = qV = (20.8×10^{-19} C)×(11.52V)= **2.40×10^{-17} J**

Problem 8a.- How much energy is necessary to put an alpha particle and a sodium nucleus 0.529×10^{-10} m apart?
The charge of an alpha particle is 3.2×10^{-19} C
The charge of a sodium nucleus is 17.6×10^{-19} C

Solution: Like the previous problem, but this time we directly get the energy.

$$P.E. = k\frac{q_1 q_2}{r} = 9\times10^{9}\frac{(3.2\times10^{-19})(17.6\times10^{-19})}{0.529\times10^{-10}} = \mathbf{9.6\times10^{-17}\ J}$$

Problem 9.- A sphere of radius R_1 has charge Q and another sphere of radius R_2 is initially uncharged. Then, they are connected through a wire, so charge flows from the first sphere to the second until the potentials are equilibrated. Calculate the charge on each sphere after reaching equilibrium.

Solution: Recall that the voltage on the surface of a sphere that contains charge Q is given by $\frac{kQ}{R}$, so after spheres 1 and 2 are at the same potential:

$$\frac{k(Q - q)}{R_1} = \frac{kq}{R_2}$$

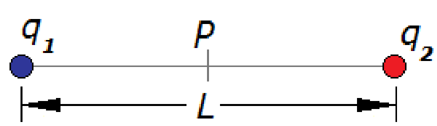
Where q is the charge transferred from sphere 1 to sphere 2, then:

$$\frac{k(Q - q)}{R_1} = \frac{kq}{R_2} \rightarrow R_2(Q - q) = R_1 q \rightarrow q = \frac{R_2}{R_1 + R_2}Q$$

The remaining charge on sphere 1 is:

$$Q - q = \frac{R_1}{R_1 + R_2}Q$$

Problem 10.- Determine the value of the electric potential at a point *P* midway between a charge q_1=-16μC and another q_2=+11.2μC that are at a distance *L*=10cm apart.

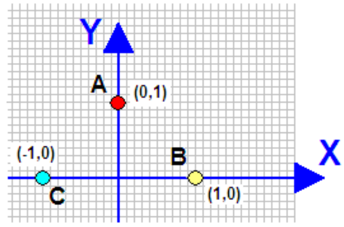


Solution: Potential is a scalar, so all we need to do is add the two contributions:

$$V_1 = k\frac{q_1}{r} = 9\times10^{9}\frac{(-16\times10^{-6})}{0.05} = \text{-}2.88\text{MV}$$
$$V_2 = k\frac{q_2}{r} = 9\times10^{9}\frac{(11.2\times10^{-6})}{0.05} = 2.016\text{MV}$$

Adding the values: V = **-864 kV**

Problem 11.- Determine the electric potential at the origin of coordinates due to the charges located at position A, B and C. Consider the coordinates given in meters and charge of q_A=-2μC, q_B=-1μC and q_C=1μC.



Solution: We can calculate the contribution from each charge to the potential at the origin:

$$V_A = k\frac{Q}{d} = 9\times10^9 \frac{-2\times10^{-6}}{1} = -18{,}000\text{V} \quad V_B = k\frac{Q}{d} = 9\times10^9 \frac{-1\times10^{-6}}{1} = -9{,}000\text{V}$$

$$V_C = k\frac{Q}{d} = 9\times10^9 \frac{1\times10^{-6}}{1} = 9{,}000\text{V/m}$$

The potential is the sum: **-18,000 V**

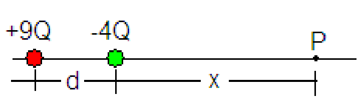
Problem 12.- A free 81μC charge is placed 1.8 mm from an identical, but fixed in space, 81μC charge. The free charge accelerates due to electric repulsion. Calculate the speed of the free charge when it is very far away if its initial velocity was zero and its mass is 2.5g

Solution: The potential energy is: $Vq = k\frac{Q}{d}q$ and this will be traded for kinetic energy, so:

$$\frac{1}{2}mv^2 = k\frac{Q}{d}q \rightarrow v = \sqrt{\frac{2kQq}{md}} = \sqrt{\frac{2\times9\times10^9\times(81\times10^{-6})^2}{0.0025\times0.0018}} = \mathbf{5{,}100\ m/s}$$

Problem 13.- Given the two charges shown in the figure:

a) At what position *x* is the electric field zero?
b) At what position *x* is the electric potential zero?
c) Find the position between the two charges where the electric potential is zero.



Solution: The electric field due to the positive charge is:

$$E_p = k\frac{9Q}{(d+x)^2}$$

this vector points towards the right.

On the other hand, the field due to the negative charge points towards the left and has a magnitude of:

$$E_n = k\frac{4Q}{x^2}$$

For the net field to be zero we need to have:

$$k\frac{9Q}{(d+x)^2} = k\frac{4Q}{x^2} \rightarrow \boldsymbol{x = 2d}$$

For the potential to be zero we need:

$$k\frac{9Q}{d+x} = k\frac{4Q}{x} \rightarrow \boldsymbol{x = 4d/5}$$

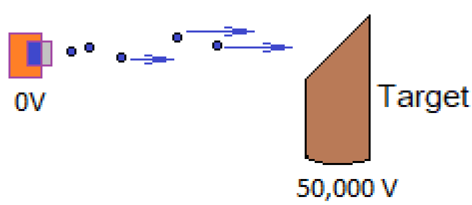
For the potential between the two charges consider the distance to the -4Q charge to be “x” then:

$$k\frac{9Q}{d-x} = k\frac{4Q}{x} \rightarrow \boldsymbol{x = 4d/13}$$

Problem 14.- If a positively charged particle enters a region of uniform electric field which is perpendicular to the particle’s initial velocity, will the kinetic energy of the particle increase, decrease or stay the same? Why?

Solution: It will increase. This is because the particle will accelerate in the direction of the field from an initial zero velocity in that direction.

Problem 15.- In an X ray machine for dentists, electrons are accelerated from a potential at zero to a maximum of 50kV. These electrons hit a target (made of copper, for example) and generate X rays. Calculate the speed of the electrons just before hitting the target.
The charge of an electron is -1.6×10^{-19} C.
The mass of an electron is 9.1×10^{-31} kg.



Solution: 50kV is already enough to start generating relativistic effects of 0.5%, but we will ignore that correction for now and use the non-relativistic equation:

$$\frac{1}{2}mv^2 = \mathrm{Vq}$$

$$\rightarrow \mathrm{v} = \sqrt{\frac{2\mathrm{Vq}}{\mathrm{m}}} = \sqrt{\frac{2\times50000\times(1.6\times10^{-19})}{9.1\times10^{-31}}} = \mathbf{1.33\times10^{8}\ m/s}$$

Problem 15a.- In analytical chemistry there are instruments known as HPLC that use a mass spectrometer where electrons (mass 9.1×10^{-31}kg, charge 1.6×10^{-19}C) are accelerated to 70V to ionize molecules. Calculate the speed of these electrons.

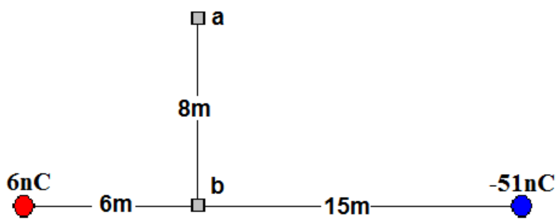
Solution: $v=\sqrt{\dfrac{2\times qV}{m}}=\sqrt{\dfrac{2\times1.6\times10^{-19}\times70}{9.1\times10^{-31}}}=$ **4.96×10^{6} m/s**

Problem 16.- In the figure we see two charges of +6nC and -51nC fixed in the indicated positions. Considering infinite to be the reference for the electric potential (V=0), calculate:

a) Voltage at "a"

b) Voltage at "b"

c) The external work needed to move a charge of -3nC from "a" to "b"



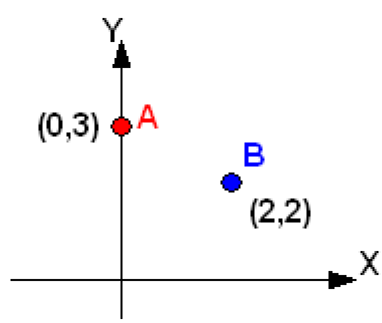
Solution:

a) $V_a=9\times10^{9}\dfrac{6\times10^{-9}}{10}-9\times10^{9}\dfrac{51\times10^{-9}}{17}=$ 5.4V - 27V = **-21.6 V**

b) $V_b=9\times10^{9}\dfrac{6\times10^{-9}}{6}-9\times10^{9}\dfrac{51\times10^{-9}}{15}=$ 9V – 30.6V = **-21.6 V**

c) Zero

Problem 17.- In this charge distribution, find the electric potential at the origin of coordinates.



Solution: We can find the electric potential at the origin by adding two scalars: A positive contribution from Q_A and a negative one from Q_B

$$V_A = k\frac{Q_A}{r} = 9\times10^9 \frac{4\times10^{-6}}{3} = 12{,}000 \text{ V.}$$

$$V_B = k\frac{Q_B}{r} = 9\times10^9 \frac{(-5\times10^{-6})}{2\sqrt{2}} = -15{,}909 \text{ V.}$$

So, the electric potential is **-3,910 V**

Problem 18.- Determine the electric potential at the origin of coordinates due to the charges located at A, B and C. Consider the coordinates given in meters and charges Q_A=4μC, Q_B=4μC and Q_C=5μC.