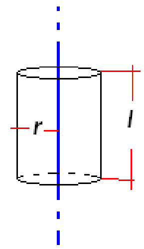
Physics II

Gauss

Problem 1.- Calculate the electric field 2cm away from a long thin wire that has a uniform linear density of charge $\lambda = 25\mu C/m$

Solution: Consider a cylinder that surrounds a short section of the wire *l*:



The charge enclosed by the cylinder is the length of the wire inside the cylinder times the linear density:

$Q = \lambda l$

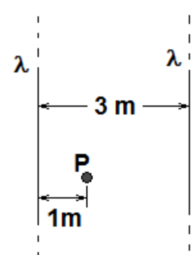
The electric field points away from the wire and perpendicular to the lateral surface of the cylinder, so Gauss's law indicates:

$$E_{\perp} Area = 4\pi kQ \rightarrow E(2\pi rl) = 4\pi k(\lambda l) \rightarrow E = \frac{2k\lambda}{r}$$

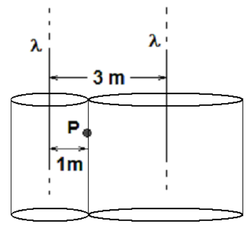
And with the values of the problem:

$$E = \frac{2k\lambda}{r} = \frac{2(9\times 10^{9})(25\times 10^{-6})}{0.02} = \mathbf{2.25\times 10^{7}\ V/m}$$

Problem 2.- There are two parallel infinite wires with linear density of charge $\lambda = 2.5\mu C/m$ separated by 3 meters.
Calculate the electric field at point P.
Suggestion: Use Gauss's theorem twice and add the *vectors.*



Solution: You can use Gauss' theorem to find the electric field produced by one infinite wire, which is $E = \frac{2k\lambda}{r}$. Here we have two wires, each one contributes an electric field calculated below.



$$E_1 = \frac{2(9\times10^9)(2.5\times10^{-6})}{1} = 45{,}000V/m$$
$$E_2 = \frac{2(9\times10^9)(2.5\times10^{-6})}{2} = 22{,}500V/m$$

One vector is to the right and the other to the left, so the sum gives **E= 22,500 V/m** to the right.

Problem 3.- Find the electric field in all space due to a spherical distribution of charge given by the density.

$$\rho = a(R-r) \quad r < R$$

Solution: We use spherical surfaces to solve this problem. The charge enclosed can be found by integration. Due to the spherical shape, we can use spherical shells as volume differentials.

For $r > R$, outside the sphere

$$E = \frac{k}{r^2}\int_0^R a(R-r)4\pi r^2 dr = \frac{\pi kaR^4}{3r^2}$$

For $r < R$, inside the sphere

$$E = \frac{k}{r^2}\int_0^r a(R-r)4\pi r^2 dr = 4\pi ka\left(\frac{Rr}{3} - \frac{r^2}{4}\right)$$

Problem 4.- A sphere of radius *R* has a charge density $\rho = Cr^3$, where *C* is a constant and *r* is the distance to center of the sphere. Find the magnitude of the electric field at a distance *r=R/2*.

Solution: According to Gauss law

$$E(Area) = \frac{Q}{\varepsilon_\circ},$$

And for a distance *r=R/2* the area is

$$Area = 4\pi\left(\frac{R}{2}\right)^2 = \pi R^2$$

The enclosed area is found by integration.

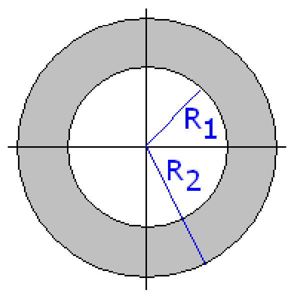
$$Q = \int_0^{R/2} \rho dV = \int_0^{R/2} Cr^3 (4\pi r^2 dr) = \frac{\pi C R^6}{96}$$

Then, the electric field is:

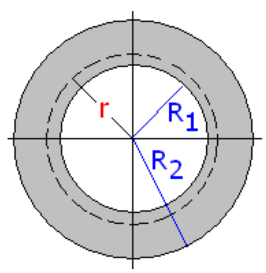
$$\pi R^2 E = \frac{\pi C R^6}{96\varepsilon_\circ} \rightarrow E = \frac{C R^4}{96\varepsilon_\circ}$$

Problem 5.- A spherical shell of internal radius R_1 and external R_2 has a constant charge density in its volume ρ.
Calculate the electric field at a distance r from the center. Consider 3 cases:

a) $r<R_1$
b) $R_1<r<R_2$
c) $r>R_2$



Solution: Inside the shell, when $r<R_1$ the electric field is zero, because the enclosed charge is zero. At a point in the shell, where $R_1<r<R_2$ we can use Gauss law, but being careful to only consider the charge between R_1 and r.



The charge is found by multiplying the density times the volume between R_1 and r. We can do this because the density is constant, otherwise we would need to divide the shell into layers (like an onion) and integrate.

The enclosed charge is: $\mathrm{Q} = \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1{}^3\right)$

And Gauss law gives us

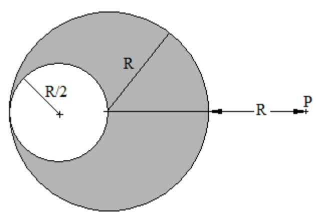
$$\mathrm{E} = \frac{\mathrm{kQ}}{r^2} = \frac{k\rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1{}^3\right)}{r^2} = \frac{4}{3}\pi k\rho\left(r - R_1{}^3 / r^2\right)$$

Finally, outside the sphere we can calculate the electric field as if all the charge where at the center of the shell. The total charge is:

$$Q = \rho\left(\frac{4}{3}\pi R_2^{\ 3} - \frac{4}{3}\pi R_1^{\ 3}\right)$$

And the electric field: $E = \frac{kQ}{r^2} = \frac{4}{3}\pi\frac{k\rho\left(R_2^{\ 3} - R_1^{\ 3}\right)}{r^2}$

Problem 6.- Find the electric field at point P due to a sphere of radius R and density of charge ρ, where a sphere of radius R/2 has been extracted, leaving that volume hollow, as shown in the figure.



Solution: By using Gauss law, each distribution of charge can be considered as a source of electric field. A larger sphere R and a smaller sphere R/2, with negative density

$$E = \frac{k\rho\frac{4}{3}\pi R^3}{(2R)^2} - \frac{k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{(2.5R)^2} = \frac{23}{75}\pi kR\rho$$