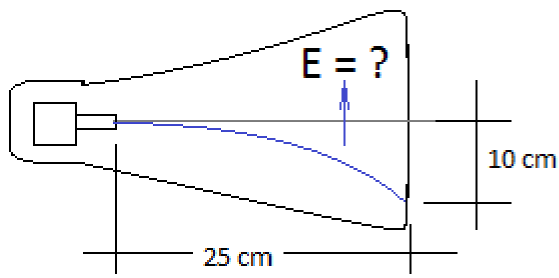
Physics II

More electric force problems

Problem 1.- In an analog oscilloscope, the screen measures 20cm vertically and the electron gun emits electrons with 15,000eV of kinetic energy, 25cm behind it.
To sweep the screen, an electric field is applied in the vertical direction. Calculate the strength of this field, if it is enough to deflect the trajectory of the electrons 10cm down as shown in the figure.



Solution: The velocity of the electrons when they leave the gun is:

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.6\times10^{-19})(15{,}000)}{9.1\times10^{-31}}} = 7.26\times10^{7}\,m/s$$

And the time to reach the screen is:

$$t = \frac{0.25m}{7.26\times10^{7}\,m/s} = 3.44\times10^{-9}\,s$$

Knowing this time and the vertical displacement, we can calculate the acceleration:

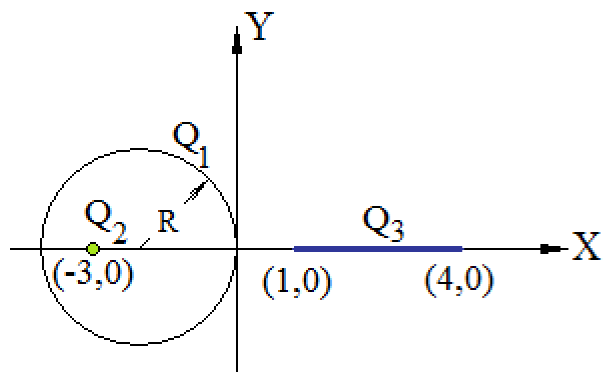
$$a = \frac{2\times d}{t^2} = \frac{2\times 0.1m}{\left(3.44\times10^{-9}\,s\right)^2} = 1.69\times10^{16}\,m/s^2$$

And the force divided by the charge gives us the strength of the electric field:

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.1\times10^{-31})(1.69\times10^{16})}{1.6\times10^{-19}} = 96{,}000\text{V/m}$$

Problem 2.- In the figure we have: A sphere with radius R=2m, uniformly charged in all its volume with Q_1 = -30μC, a point charge of value Q_2= 10μC inside the sphere at position (-3,0) and a 3m long wire located on the X-axis with total charge Q_3 = -20μC uniformly distributed over its length. Calculate

a) The electric force over Q_2 due to Q_1.
b) The electric force over Q_2 due to Q_3.
c) The electric force over Q_1 due to Q_3.



Solution:

a) Due to the geometry of the problem, the charge in the sphere that is beyond 1m from its center does not apply a net force on Q_2 due to symmetry. Only the charge inside a 1m radius will produce a net force. That charge is 1/8 of the total charge, -30/8 μC and it can be taken as concentrated at the center of the sphere. Then, the force on Q_2 will be

$$F = k\frac{Q_1Q_2}{d^2} = 9\times10^9\,\frac{(30/8\times10^{-6})(10\times10^{-6})}{1^2} = \mathbf{0.338\ N}$$

Since the charges have opposite signs, the force will be attractive, so to the right in this case.

b) To calculate the force due to the wire, we divide it in on short differentials of charge and integrate

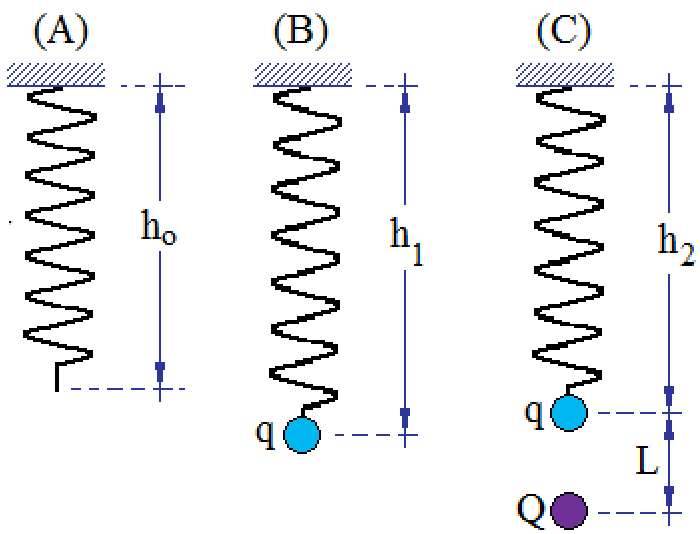
$$F = \int k\frac{Q_2}{d^2}dQ_3 = kQ_2\frac{Q_3}{3}\int_4^7\frac{dx}{x^2} = k\frac{Q_2Q_3}{4\times7} = 9\times10^9\,\frac{(10\times10^{-6})(20\times10^{-6})}{4\times7} = \mathbf{0.064\ N}$$

Again, the charges have opposite signs, so the force is attractive and to the right in this case.

c) This is like the previous case (b), where we can imagine all the charge of the sphere located at its center:

$$F = \int k \frac{Q_1}{d^2} dQ_3 = kQ_1 \frac{Q_3}{3} \int_3^6 \frac{dx}{x^2} = k \frac{Q_1 Q_3}{3 \times 6} = 9 \times 10^9 \frac{(30 \times 10^{-6})(20 \times 10^{-6})}{3 \times 6} = \mathbf{0.3\ N}$$

Problem 3.- Figure (A) shows a spring with un-stretched length $h_o = 0.12$m. Then you hang a mass m = 0.002 kg with charge q = 1μC and the spring stretches as shown in (B) reaching equilibrium with a length $h_1 = 0.14$m. Finally, a charge Q is place below q as shown in (C) and the new equilibrium length is $h_2 = 0.13$m with a charge separation L = 0.045m.
Calculate:
a) The spring constant k_r
b) The force charge Q produces on q in figure (C)
c) The value of charge Q



Solution:

a) The spring stretches 0.02m with a weight 0.02N, so k_r = **1N/m**.

b) In (C), when q is in equilibrium you have its weight 0.02N pulling down and the spring force is only solo 0.01N, so the electric force must be **0.01N upwards**.

c) Since the force is repulsive Q and q have the same sign. To find the value we solve:

$$F = k \frac{Qq}{L^2}$$

Obtaining

$$Q = \frac{FL^2}{kq} = \frac{0.01 \times 0.045^2}{9 \times 10^9 \times 1 \times 10^{-6}} = \mathbf{2.25nC}$$