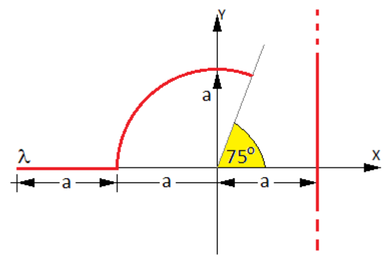
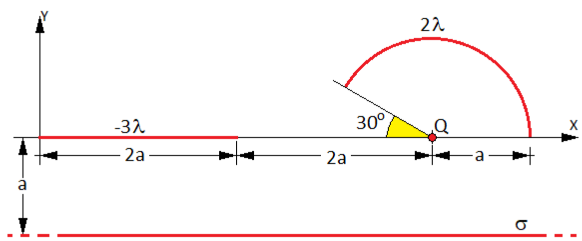
Physics II

More electrostatics problems

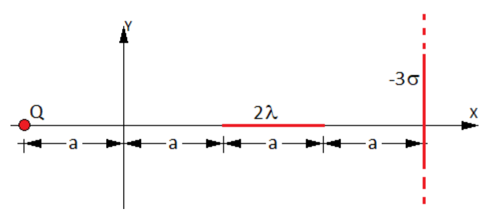
Problem 1.- Find the electric field at the origin of coordinates due to the objects shown in the figure and describe below.
a) A segment of wire of linear charge density λ
b) A wire in the shape of an arc of a circle with radius a, and linear charge density λ.
c) An infinite straight wire with linear charge density λ.



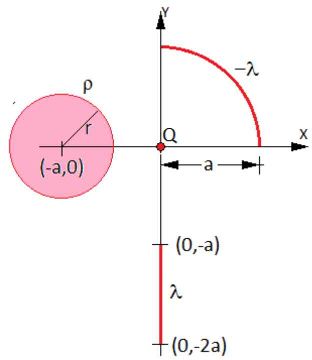
Problem 2.- Find the electric force on the point charge Q (+) located at (4a, 0), due to:
a) The 2a-long wire shown in the figure with linear charge density -3λ
b) A wire in the shape of an arc with radius a, and linear charge density 2λ
c) An infinite plane of surface charge density σ parallel to the xz plane



Problem 3.- You have an infinite plane with surface charge density -3σ, a point charge Q and a wire with linear charge density 2λ. Calculate:
a) The electric force on the point charge.
b) The electric force on the wire.

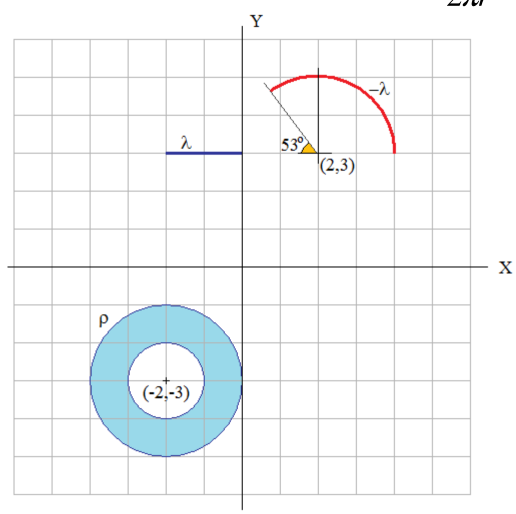


Problem 4.- The figure shows a sphere with radius r and uniform charge density ρ, a wire in the shape of an arc with linear charge density –λ, a straight wire with linear charge density λ and a point charge Q. Find the electric force on Q.

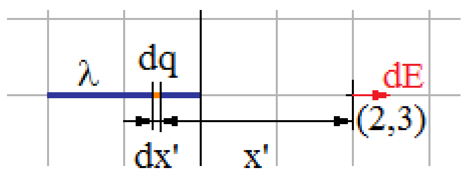


Problem 5.- Find the electric field at point (2,3) due to

a) The straight wire of linear density λ
b) The wire in the shape of an arc with linear charge density –λ
c) The hollow sphere whose charge density for r = [1 2] is $\rho = \dfrac{\rho_o}{2\pi r^2}$



Solution:
a) Since the straight wire has positive charge, the field on (2,3) will be directed to the right.
To calculate its value, we divide the wire in short differential lengths dx' with charge $\lambda dx'$ and call the distance to (2,3) x', as shown in the diagram:



The electric field differential is

$$dE = k\frac{dq}{x'^2} = k\frac{\lambda dx'}{x'^2}$$

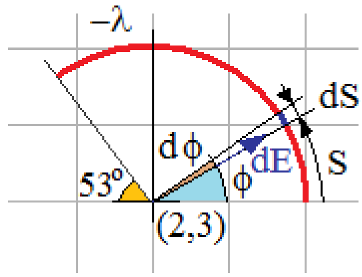
To find the total value we integrate from 2m to 4m, which are the limits of x'

$$E = \int_2^4 k\frac{\lambda dx'}{x'^2} = \frac{k\lambda}{4}$$

And the vector is

$$\vec{E} = \left(\frac{k\lambda}{4}, 0\right)$$

b) For the wire bent as an arc, we follow a similar procedure. We divide the wire in differentials of charge dq. Here however, we consider that the vector changes direction, so we integrate the components x and y separately.



The length differential is $dS = Rd\phi = 2d\phi$ and the charge differential $dq = -2\lambda d\phi$

The sign of the charge indicates that the vector at (2, 3) should be directed towards the charge and the magnitude is

$$dE = k\frac{dq}{R^2} = k\frac{2\lambda d\phi}{R^2} = k\frac{2\lambda d\phi}{2^2} = k\frac{\lambda d\phi}{2}$$

We decompose this vector in two parts:

$$dE_x = k\frac{\lambda d\phi}{2}\cos\phi$$
$$dE_y = k\frac{\lambda d\phi}{2}\sin\phi$$

By integrating we find:

$$E_x = \int_0^{127^o} \frac{k\lambda d\phi}{2}\cos\phi = \frac{k\lambda}{2}(\sin 127^o - \sin 0^o) = \frac{k\lambda}{2}(0.8)$$

$$E_y = \int_0^{127^o} \frac{k\lambda d\phi}{2}\sin\phi = \frac{k\lambda}{2}(-\cos 127^o + \cos 0^o) = \frac{k\lambda}{2}(1.6)$$

The final vector is

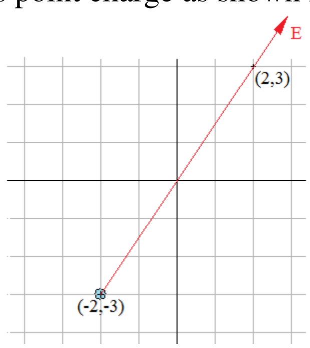
$$\vec{E} = (0.4, 0.8)k\lambda$$

c) For the hollow sphere, we can use Gauss' theorem and consider as if all its charge were located at its center. We find the total charge by integration:

$$Q = \int_{R1}^{R2} \rho dV = \int_1^2 \frac{\rho_o}{2\pi r^2} 4\pi r^2 dr = \int_1^2 2\rho_o dr = 2\rho_o$$

Notice that we used the volume differential $dV = 4\pi r^2 dr$, which is the same as dividing the sphere in layers, like an onion.

We replace the hollow sphere by this point charge as shown in the diagram.



The magnitude of the electric field vector is

$$E = k\frac{Q}{d^2} = 9\times10^{-9}\frac{2\rho_o}{4^2+6^2} = \frac{9}{26}\times10^{-9}\rho_o$$

Its two components are:

$$E_x = \frac{9}{26}\times10^{-9}\rho_o\frac{4}{\sqrt{52}}$$

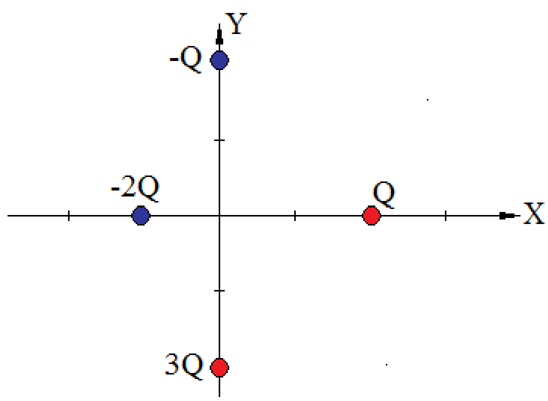
$$E_y = \frac{9}{26}\times10^{-9}\rho_o\frac{6}{\sqrt{52}}$$

Then the vector will be

$$\vec{E} = \frac{9}{13\sqrt{52}} \times 10^{-9} \rho_o (2,3)$$

Problem 6.- The figure shows four charges:
-Q at (0,2m),
+Q at (2m,0),
-2Q at (-1m,0) and
3Q at (0,-2m).
Calculate:

a) The horizontal component of the electric field (E_x) at the origin of coordinates (0, 0)
b) The vertical component of the electric field (E_y) at the origin of coordinates (0, 0)
c) The magnitude of the electric field at the origin of coordinates (0, 0)
Answer in terms of Q and k (9×10^9 Nm^2/C^2)



Solution:

a) Only charges -2Q and +Q produce horizontal electric field at the origin. The value is

$$E_x = -k\frac{2Q}{1^2} - k\frac{Q}{2^2} = \mathbf{-2.25}\, kQ$$

Notice that the minus sign indicates the direction of the vector, being both contributions to the left.

b) In the vertical direction the charges to consider are –Q and 3Q, so

$$E_y = k\frac{3Q}{2^2} + k\frac{Q}{2^2} = \mathbf{1}\, kQ$$

c) And the magnitude of the vector is:

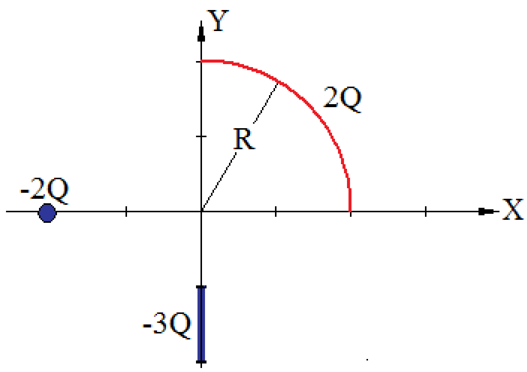
$$E = \sqrt{2.25^2 + 1^2}\, kQ = \mathbf{2.46}\, kQ$$

Problem 7.- The figure shows
- A point charge –Q at (-2m,0)
- An 90-degree arc of radius R=2m centered at the origin of coordinates and with uniformly distributed charge over its length equal to +2Q
- A 1-meter wire located between the points (0,-1m) and (0,-2m) with charge -3Q distributed uniformly over its length.

Find the electric field at the origin due to

a) The point charge.
b) The 90-degree arc.
c) The straight wire.

Respond in terms of Q and k (9×10^9 Nm^2/C^2)



Solution:

a) The point charge will produce an electric field to the left, so it only has a horizontal component:

$$E_x = -k\frac{2Q}{2^2} = \mathbf{-1}\,kQ$$

While in the vertical direction:

$$E_y = \mathbf{0}$$

b) We can divide the arc in differentials and then we integrate the components. Due to the symmetry of the problem, it is only necessary to do this for one component, as the other is the same.

$$dE_x = k\cos\theta\frac{dQ}{d^2}$$

$$\to E_x = -k\frac{2Q}{\pi/2}\int_0^{\pi/2}\frac{\cos\theta d\theta}{2^2} = -\frac{kQ}{\pi}\ \mathbf{= -0.318}\,kQ \text{ and } E_y = -\frac{kQ}{\pi}\ \mathbf{= -0.318}\,kQ$$

b) For the case of the straight wire, we divide it in length differentials and integrate. In this case the horizontal component is zero, so

$E_x = \mathbf{0}$

But the vertical component is downwards with a value

$$E_y = -k\frac{3Q}{1}\int_1^2 \frac{dy}{y^2} = -\frac{k3Q}{2\times 1} = \mathbf{-1.5}\,kQ$$

Problem 8.- You have three identical charges $q = 1\mu C$ located at the corners of an equilateral triangle with side d = 0.2 m. Calculate the force on one of the charges.

Solution: The force produced by any one charge on another is:

$$F = k\frac{q^2}{d^2} = 9\times 10^9\,\frac{Nm^2}{C^2}\,\frac{(1\times 10^{-6}C)^2}{(0.2m)^2} = 0.225N$$

When adding two of them we consider that the angle between them is 60º, then:

$$F = \sqrt{0.225^2 + 0.225^2 + 2\times\cos 60^\circ \times 0.225\times 0.225}$$
$$F = 0.389N$$

Problem 9.- You have a cylinder with total charge $Q = 1nC$ uniformly distributed over all its volume. Radius $R = 0.1m$, height $h = 4m$ and you want to find the electric field at a point 10m below its base.

Solution: Since the cylinder is so narrow, it can be approximated by a wire with linear charge density:

$$\lambda = \frac{1nC}{4m} = 0.25nC/m$$

We divide its height in small disks and calculate the electric field contributed by this differential of charge.

$$dE = \frac{1}{4\pi\varepsilon_\circ}\frac{\lambda dz}{(10-z)^2}$$

Integrating

$$E = \frac{\lambda}{4\pi\varepsilon_\circ}\int_0^h \frac{dz}{(10-z)^2} = \frac{\lambda}{4\pi\varepsilon_\circ}\left(\frac{1}{(10-h)} - \frac{1}{10}\right)$$

And with the values given $\quad E = 0.15V/m$