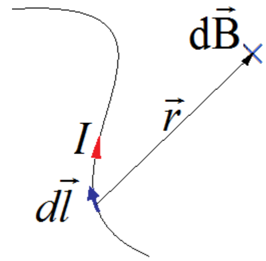
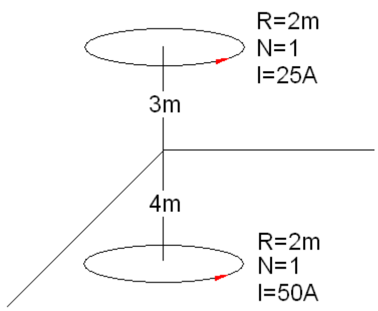
Physics II

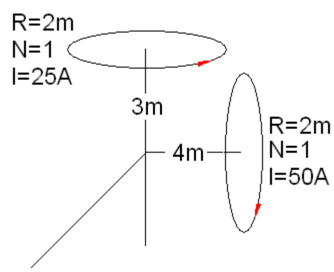
Biot and Savart



$$\mathrm{d}\vec{\mathrm{B}} = \frac{\mu_o I d\vec{l} \times \vec{r}}{4\pi r^3}$$

Problem 1.- Use the results of Biot and Savart law to find the magnetic field produced at the origin by the sets of coils shown in the figures:





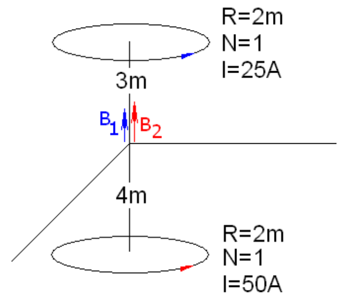
Solution: In the first case, we have two contributions to the magnetic field vector:

$$\mathrm{B}_1 = \frac{\mu_o I_1 R^2}{2(R^2 + x^2)^{3/2}} = \frac{4\pi\times10^{-7}(25)(2^2)}{2(2^2+3^2)^{3/2}} = 1.34\times10^{-6} T$$

$$\mathrm{B}_2 = \frac{\mu_o I_2 R^2}{2(R^2 + x^2)^{3/2}} = \frac{4\pi\times10^{-7}(50)(2^2)}{2(2^2+4^2)^{3/2}} = 1.40\times10^{-6} T$$

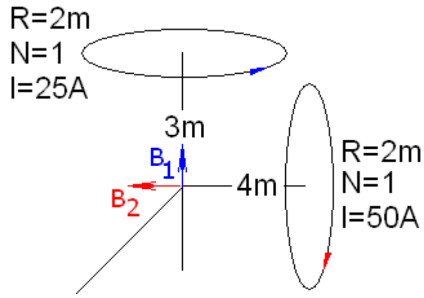
Since the vectors point in the same direction, we just need to add their values to find their sum:

$$\mathrm{B} = \mathrm{B}_1 + \mathrm{B}_2 = 2.74\times10^{-6}\mathrm{T}$$

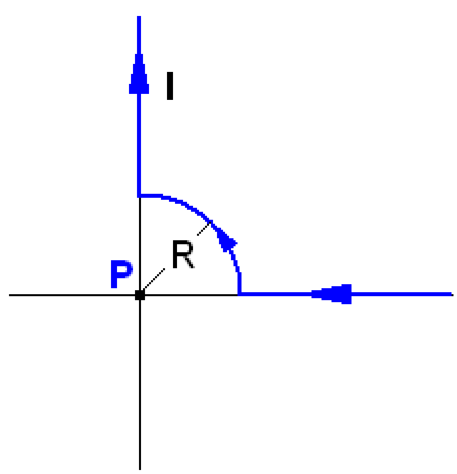


In the second case, the magnitudes of the two vectors are the same as before, but now they make a right angle, so to add them we will use the Pythagoras theorem:

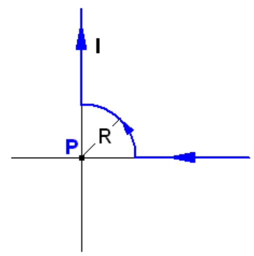
$$B = \sqrt{B_1^2 + B_2^2} = 1.94 \times 10^{-6}\,\mathrm{T}$$



Problem 2.- Calculate the magnetic field at point "P" due to the wire shown in the figure, which transports a current I.



Solution: You can use Biot and Savart's Law to find the magnetic field:



$$B = \int \frac{\mu_o I d\vec{l} \times \hat{r}}{4\pi r^2}$$

In the integral, we only need to include the arc (section of circle). This is because the straight wires contribute zero to the magnetic field since the angle between the vectors in the integral is 180º or 0º.

The arc is at a constant distance R from P and the vectors $d\vec{l}$ and $\hat{r}$ are always at 90º, so their product is just the product of their magnitudes times the unit vector perpendicular to both.

Then:

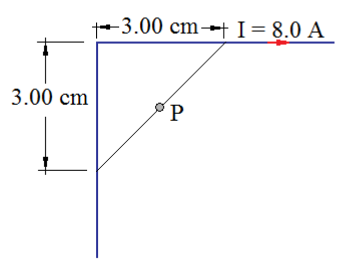
$$B = \int \frac{\mu_o I d\vec{l} \times \hat{r}}{4\pi r^2} = \frac{\mu_o I \hat{k}}{4\pi R^2} \int dl$$

Where $\hat{k}$ is a unit vector coming from the figure towards us.

Since the sector is ¼ of a circle: $\int dl = \frac{2\pi R}{4}$ and the magnetic field is then:

$$B = \frac{\mu_o I \hat{k}}{4\pi R^2} \frac{2\pi R}{4} = \frac{\mu_o I \hat{k}}{8R}$$

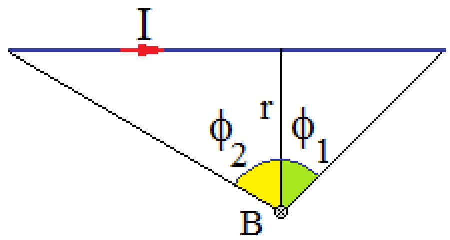
Problem 3.- Calculate the magnetic field at P (in the middle of the segment shown) produced by the long wire that carries 8.0 A.



Solution: We can consider each part of the wire separately, and using a previous result for a finite wire:

$$B = \frac{\mu_o I}{4\pi r}\left(\sin\varphi_1 + \sin\varphi_2\right)$$

Where the angles are indicated in the figure below.



In our case, the wire at the top of the figure makes one angle of 45° and another of 90°, and the distance r is 1.5cm. The vertical wire in the figure contributes the same amount and in the same direction (towards the figure). Then, the magnetic field is:

$$B = 2\frac{\mu_o I}{4\pi r}\left(\sin 45^o + \sin 90^o\right)$$

$$B = 2\frac{4\pi\times10^{-7}(8)}{4\pi(0.015)}\left(\frac{\sqrt{2}}{2}+1\right) = \frac{10^{-4}(16)}{15}\left(\frac{\sqrt{2}}{2}+1\right) = \mathbf{1.82\times10^{-4}\ T}$$

Problem 4.- In an ID circuit you have a conductor in the shape of a right triangle as shown in the figure. Find the magnetic field in direction and magnitude at point P if the circulating current is 2.4mA.