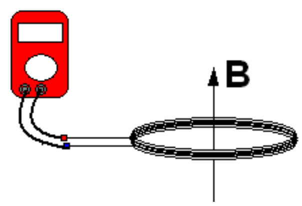
Physics II

Induced Electromotive Force

Faraday, Lenz's law: $emf = -\frac{\Delta\phi}{time}$, where $\phi = \text{NBA}\sin\angle^{\text{B}}_{\text{Surface}}$

Problem 1.- A coil has a diameter of 0.25m and it is made of 1,500 loops of copper wire. It is in a region where the magnetic field is 5.5×10^{-5}T and perpendicular to the plane of the coil as shown in the figure. Find the average electromotive force produced by flipping the coil in a time of 0.15s, so the magnetic flux changes to the opposite direction.



Solution: The magnetic flux before flipping the coil is $\phi_{before} = NBA = 1500(5.5\times10^{-5}T)\left(\frac{\pi(0.25m)^2}{4}\right)$

= 0.00405 Tm^2

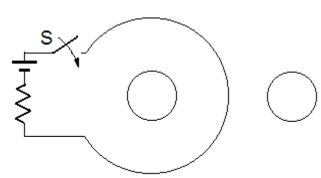
After flipping the coil, the flux will be the same in magnitude, but pointing in the opposite direction, so the change will be twice the value calculated above:

$\Delta\phi$ = -2×(0.00405 Tm^2)=-0.0081 Tm^2

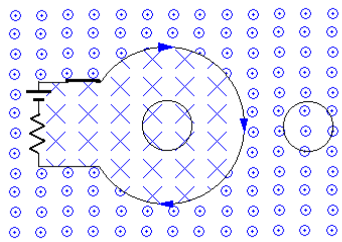
To get the electromotive force we divide by the time it took to change the flux:

$$emf = -\frac{\Delta\phi}{time} = \frac{0.0081Tm^2}{0.15s} = $$ **0.054 volts**

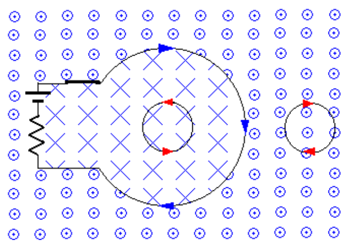
Problem 2.- What will be the direction of the current induced in each of the small circular loops when the switch "S" is suddenly closed?



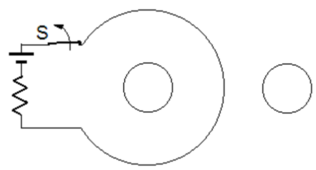
Solution: Before closing the switch, you have zero current and zero magnetic field. When you close it, the circuit the current in the main loop will create magnetic field lines as shown:



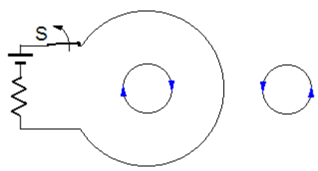
Due to Lenz's law, the induced current in the loops will try to oppose the change, so they will have the direction indicated in the figure:



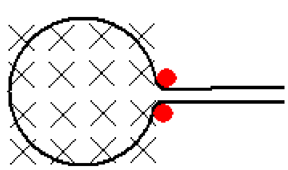
Problem 2a.- What will be the direction of the current induced in each of the small circular loops when the switch "S" is suddenly opened?



Solution: We use Lenz's law to determine the direction of the induced *emf*.



Problem 3.- A circular loop of wire of diameter 1.05m is in a region where the magnetic field is 0.225T and perpendicular to the plane of the loop. You pull the wire reducing the diameter of the loop to 0.95m in 5 seconds. Find the average *emf* produced by this change.



Solution: The *emf* is calculated using Faraday's law:

$$emf = -\frac{\Delta NBA\sin\theta}{time},$$

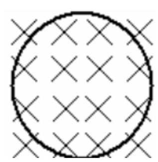
with the values of the problem:

$$emf = -\frac{NB}{time}\left(A_{final} - A_{initial}\right) = \frac{NB}{time}\left(\frac{\pi D_{initial}^2}{4} - \frac{\pi D_{final}^2}{4}\right)$$

$$= \frac{\pi(1)(0.225T)}{4(5s)}\left((1.05m)^2 - (0.95m)^2\right) = \mathbf{7.1mV}$$

Problem 3a.- 0.4 meters of wire form a square in a region with a uniform magnetic field of 0.06 Tesla perpendicular to the square. The wire is reduced in length by 0.01 m in 10 seconds keeping its square shape. Calculate the resulting *emf*.

Problem 4.- A circular loop of wire encloses an area of $1.05m^2$ and is in a region where the magnetic field has an intensity in tesla $B=0.025t^2$, where "t" is the time in seconds. The field is perpendicular to the plane of the loop as shown in the figure. Find the induced *emf* as a function of time.



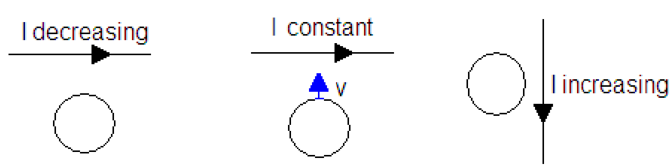
Solution: The magnetic flux is given by:

$$\phi = NBA\sin\theta = 1\times 0.025t^2\left(1.05m^2\right)\sin 90^\circ = 0.02625t^2$$

The induced *emf* can be calculated using Faraday's law, taking the derivative of the flux:

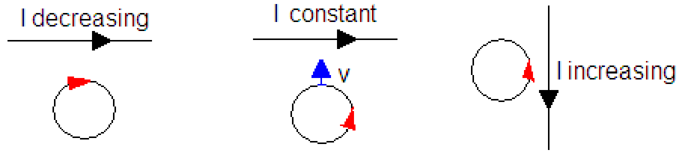
$$emf = -\frac{d\phi}{dt} = -\frac{d\left[0.02625t^2\right]}{dt} = \mathbf{0.0525\ t}$$

Problem 5.- Indicate the direction of the induced current in the hoop in each of the following cases:

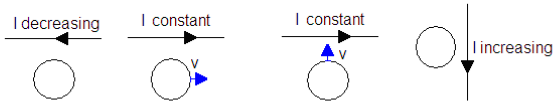


Note: In the second case the hoop is moving towards the wire.

Solution: Direction of the induced current:

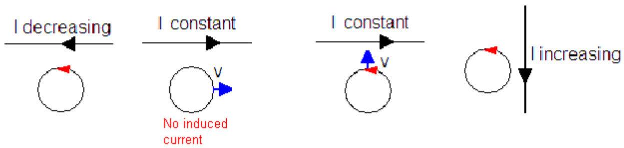


Problem 5a.- Indicate the direction of the induced current in the loop in each of the following cases:



Note: v indicates velocity.

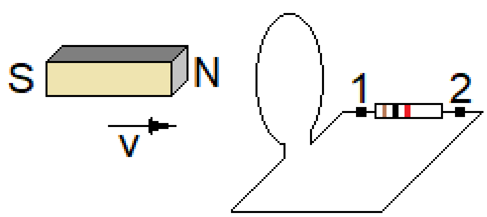
Solution: In each case we determine the induced current by using Lenz's law, that is the induced current will oppose the change in magnetic flux through the loop. The red arrow indicates the answer in each case:



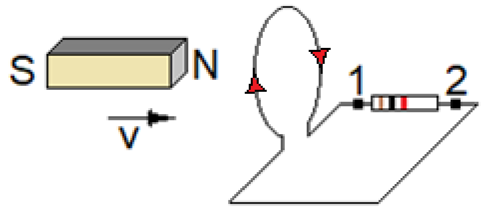
Problem 6.- How is the magnetic stripe in your credit card read when you swipe it at a store?

Solution: When you swipe your credit card, the magnetic stripe produces flux reversals on the reader, which encode your personal information. Notice that you have to move the magnetic stripe to produce induced currents.

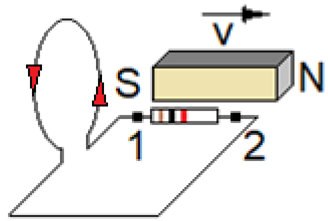
Problem 7.- A permanent magnet with its poles oriented as shown in the figure is moved towards a wire in the shape of a circle at a constant velocity. The magnet passes though the loop and moves away. The circular wire is connected to a circuit with a resistance R.
Graph and justify the variation in induced current in the resistance in terms of the time when the magnet approaches, passes through, and moves away from the loop.



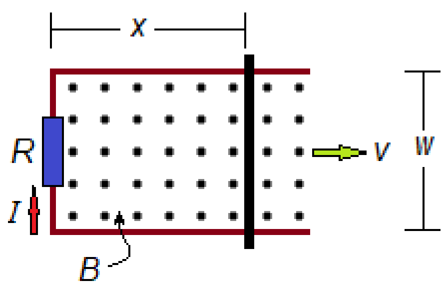
Solution: When the magnet approaches the loop, the magnetic flux will increase with more field lines going through from left to right. This will produce an induced current in the resistance from 1 to 2 as shown in the figure:



Later, when the magnet is moving away, the flux will decrease, and the current will be inverted as shown below.



Problem 8.- A circuit is built by connecting a resistance R=2Ω to a conducting wire in the shape of a U with width w = 0.5 m, and a sliding conducting bar that closes the circuit. Consider that this circuit is in a region where the magnetic field is B = 1 T perpendicular to the plane of the circuit and the resistances of the wire and bar are negligible.



a) Calculate the *emf* if the bar moves to the right at a speed v = 10m/s
b) Calculate the induced current in case (a).
c) Calculate the speed v necessary to induce a current of 0.5 A

Solution:

a) To calculate the *emf* we use Faraday's law

$$emf = -\frac{d\varphi}{dt}$$

Where the flux is

$$\phi = NBA \sin \angle_B^A = (1)(1)(wx)\sin 90^\circ$$

Given that N=1, B=1 T, the area is A=*wx,* and the angle between the surface and B is 90º.

When taking the derivative, the only variable in time is *x,* whose derivative is the velocity, so:

$$emf = -\frac{d\phi}{dt} = -wv = -0.5v$$

The negative sign indicates opposition to change in flux. The answer is **5 V**.

b) Using Ohm's law $I = \frac{V}{R} = \frac{5}{2} =$ **2.5A**

c) If the current is 0.5A the voltage is V=IR=1 V and because $emf = -0.5v$, the velocity is **2m/s**.

Note: When doing these calculations, we are neglecting the field produced by the induced current itself. A more accurate calculation should take that into account. However, in this case it would produce a small correction.