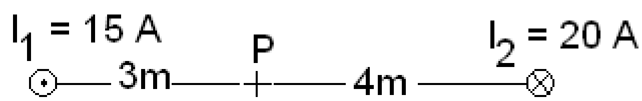
Physics II

Magnetic Field Production

Magnetic field produced by a long wire: $B = \frac{\mu_o I}{2\pi r}$, where $\mu_o = 4\pi \times 10^{-7} Tm/A$ and r is the distance to the wire.

Problem 1.- Find the magnetic field at point "P" produced by the two long straight current carrying wires shown in the figure. Answer with magnitude and direction.

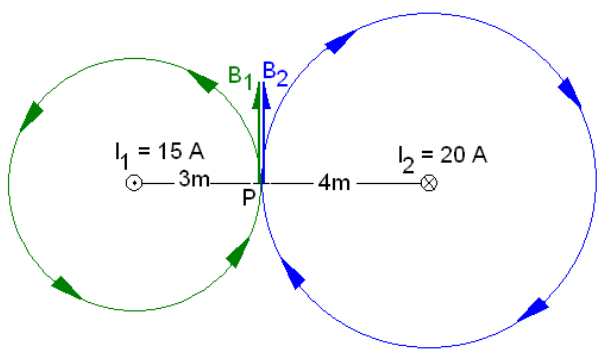


Solution: We can calculate the magnetic field produced by each current at point "P" using Ampere's law:

$$B_1 = \frac{\mu_o I_1}{2\pi r} = \frac{4\pi \times 10^{-7}(15A)}{2\pi(3m)} = 1\mu T$$

$$B_2 = \frac{\mu_o I_2}{2\pi r} = \frac{4\pi \times 10^{-7}(20A)}{2\pi(4m)} = 1\mu T$$

The right-hand rule indicates that the two vectors point upwards as shown in the figure:



The sum of the two vectors is **2μT**, upward direction.

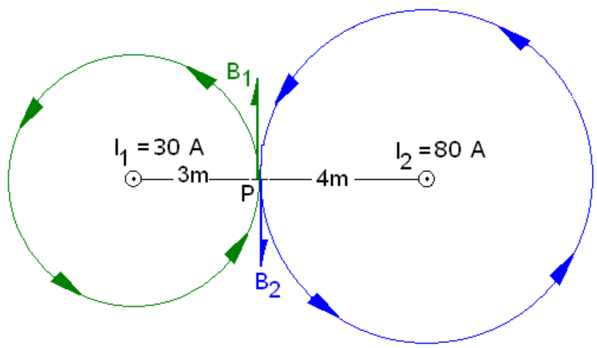
Problem 1a.- Find the magnetic field at point "P" produced by the two long straight current carrying wires shown in the figure. Answer with magnitude and direction.

Solution: We can calculate the magnetic field produced by each current at point "P" using Ampere's law:

$$B_1 = \frac{\mu_o I_1}{2\pi r} = \frac{4\pi \times 10^{-7}(30A)}{2\pi(3m)} = 2\mu T$$

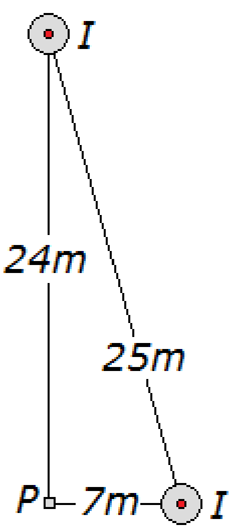
$$B_2 = \frac{\mu_o I_2}{2\pi r} = \frac{4\pi \times 10^{-7}(80A)}{2\pi(4m)} = 4\mu T$$

The right-hand rule indicates that B_1 points upwards and B_2 points downwards as shown in the figure:

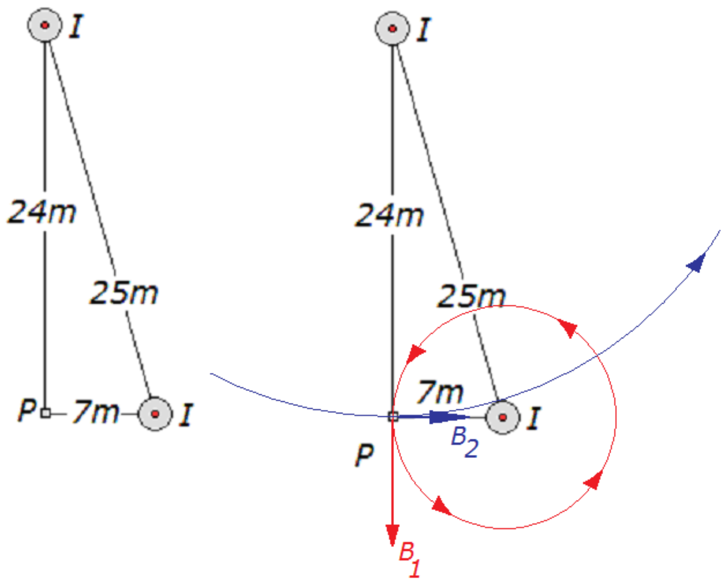


The sum of the two vectors is **2μT**, downward direction.

Problem 2.- Two long thin parallel wires are separated 25m and carry currents I=150A in the same direction. Calculate the magnetic field at a point P located 24m from one wire and 7m from the other.



Solution: Each wire will generate a magnetic field vector at point P.



Magnetic field produced by a long wire: $B = \frac{\mu_o I}{2\pi r}$, where $\mu_o = 4\pi \times 10^{-7}$

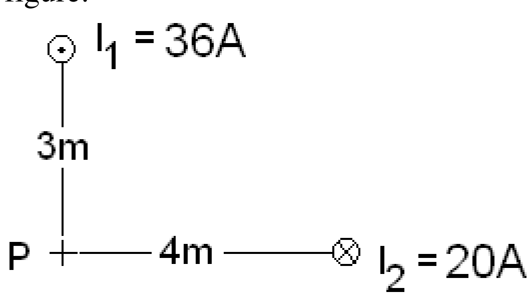
For the wire that is 24 meters away: $B_1 = \frac{\mu_o I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 150}{4\pi \times 24} = 1.25\ \mu T$

For the wire that is 7 meters away: $B_2 = \frac{\mu_o I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 150}{4\pi \times 7} = 4.28\ \mu T$

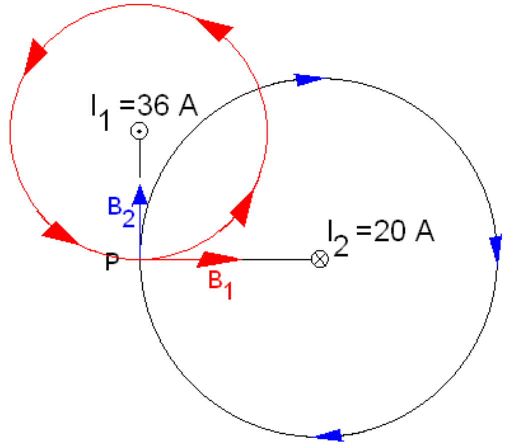
Now we need to add the vectors. To do that we notice that they make an angle of 90º, so adding them requires Pythagoras theorem:

$B = \sqrt{B_1^{\ 2} + B_2^{\ 2}} = \sqrt{1.25^2 + 4.28^2}\,\mu T = \mathbf{4.45\ \mu T}$

Problem 2a.- Find the magnetic field at point "P" produced by the two long straight current carrying wires shown in the figure:



Solution: Each wire produces a magnetic field at point P as follows:



The magnitudes of the vectors are:

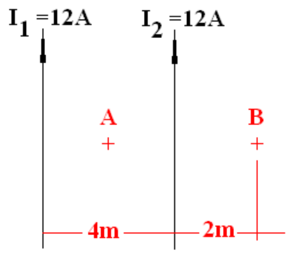
$$B_1 = \frac{\mu_o I_1}{2\pi r_1} = \frac{4\pi\times10^{-7}(36)}{2\pi(3)} = 2.4\times10^{-6}T$$

$$B_2 = \frac{\mu_o I_2}{2\pi r_2} = \frac{4\pi\times10^{-7}\frac{Tm}{A}(20)}{2\pi(4)} = 1\times10^{-6}T$$

Since the vectors are at 90º, to add them we calculate:

$$B = \sqrt{B_1^2 + B_2^2} = \mathbf{2.6\times10^{-6}\ T}$$

Problem 3.- Calculate the magnetic field at points A and B produced by the long parallel wires shown in the figure. Point A is in the middle of the two wires.



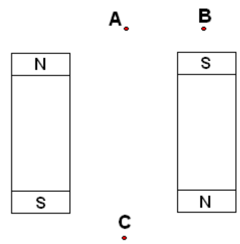
Solution: The magnetic field at point A is **zero**.
At point B there are two contributions:

Due to I_1: $B = \frac{\mu_o I_1}{2\pi r} = \frac{4\pi\times10^{-7}\times12}{2\pi(6)} = 4\times10^{-7}Tesla$

Due to I_2: $B = \frac{\mu_o I_2}{2\pi r} = \frac{4\pi\times10^{-7}\times12}{2\pi(2)} = 12\times10^{-7}Tesla$

The two vectors point in the same direction, so they add to **16×10^{-7} tesla**

Problem 4.- Indicate the ***direction*** of the magnetic field at points **A**, **B** and **C** due to the two identical bar magnets.



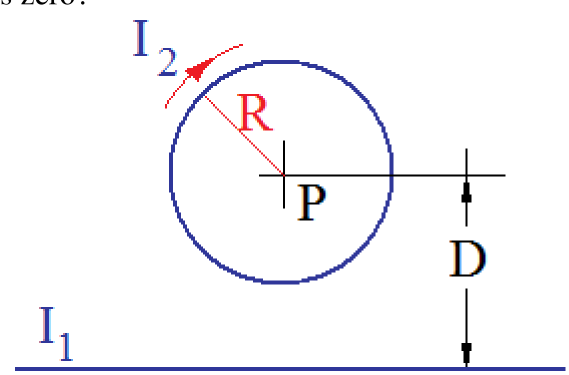
Problem 5.- Indicate if the following quantities are vectors or scalars and the units used to measure them:

(i) Electric potential	Vector or scalar? ________	Units? __________
(ii) Electric field	Vector or scalar? ________	Units? __________
(iii) Magnetic field	Vector or scalar? ________	Units? __________

Solution:

(i) Electric potential	Scalar, measured in volts or J/C.
(ii) Electric field	Vector, measured in V/m or N/C.
(iii) Magnetic field	Vector, measured in tesla (T) or gauss (G).

Problem 6.- What must be the direction and magnitude of the current I_1 in the long straight wire if the magnetic field at P is zero?



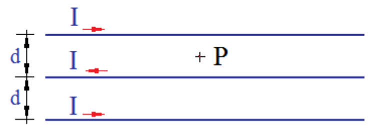
Solution: The direction of I_2 is such that at P it produces a magnetic field towards the figure. The current I_1 must flow to the right to counteract I_2. We equal the two magnitudes to get zero.

$$B_{wire} = \frac{\mu_o I_1}{2\pi D}$$

$$B_{loop} = \frac{\mu_o I_2}{2R}$$

$$\rightarrow I_1 = \frac{\pi D}{R} I_2$$

Problem 7.- Three long wires carry the currents shown in the figure below. Calculate the magnetic field at P, which is the middle point between the two top wires. And calculate the magnetic force per unit length on the top conductor.



Solution: If we call the magnetic field at P produced by the wires B_1, B_2 and B_3, their magnitudes are

$$B_1 = \frac{\mu_o I}{2\pi d/2}$$
$$B_2 = \frac{\mu_o I}{2\pi d/2}$$
$$B_3 = \frac{\mu_o I}{2\pi 3d/2}$$

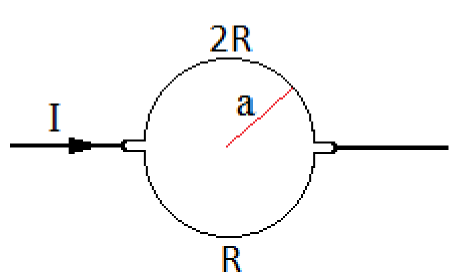
But we notice that B_1 y B_2 point towards the figure, while B_3 points away from it, so the total field is $B_1 = \frac{5\mu_o I}{3\pi d}$ towards the figure.

To find the force per unit length on the top cable, we notice that the field at its position is due to wires 2 and 3

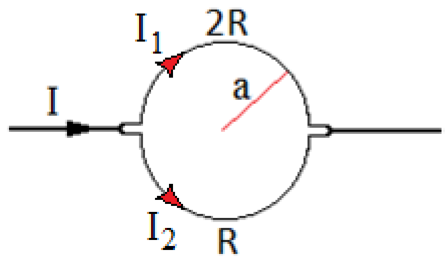
$B = \frac{\mu_o I}{2\pi d} - \frac{\mu_o I}{2\pi 2d} = \frac{\mu_o I}{4\pi d}$ towards the figure.

The force per unit length is $\frac{F}{l} = \frac{\mu_o I^2}{4\pi d}$ directed upwards.

Problem 8.- Two wires are bent in the shape of semicircles of radius a as shown below. If the top wire has a resistance 2*R* and the bottom one *R*, find the magnetic field at the center in terms of the total current I.



Solution: Since the resistance at the top is higher, the current will be lower. We can use Ohm's law to calculate the currents. If we call them I_1 and I_2 as indicated:



Ohm's law means that $2RI_1 = RI_2$, which means that I_1 is half I_2. And the sum is I, so the currents are $I_1 = I/3$ and $I_2 = 2I/3$.

With the values of the currents, we calculate the magnetic field produced by the semicircles at P:

$$B_1 = \frac{1}{2}\frac{\mu_\circ I_1}{2a} = \frac{1}{2}\frac{\mu_\circ I/3}{2a} = \frac{\mu_\circ I}{12a}$$
$$B_2 = \frac{1}{2}\frac{\mu_\circ I_1}{2a} = \frac{1}{2}\frac{\mu_\circ 2I/3}{2a} = \frac{\mu_\circ I}{6a}$$

Notice that we used half the value of a circle for each semicircle.
When adding, the vector produced by the top semicircle is towards the figure and the one from the bottom is out of the figure. Taking this into account the magnetic field at P is

$B = \frac{\mu_\circ I}{12a}$, out of the figure.