Physics II

Magnetic Force

Force on a charge moving in a magnetic field: $F = Bqv\sin\angle_{B}^{v}$

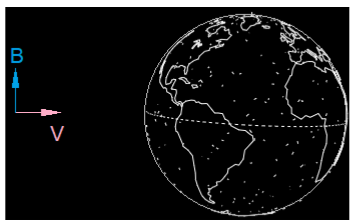
$\vec{F} = q\vec{v}\times\vec{B}$

Force on a wire carrying a current I: $F = BIl\sin\angle_{B}^{\ell}$

$\vec{F} = I\vec{l}\times\vec{B}$

Magnetic field produced by a long wire: $B = \frac{\mu_o I}{2\pi r}$, where $\mu_o = 4\pi\times10^{-7}\frac{Tm}{A}$

Problem 1.- Find the magnetic force on a proton coming towards our planet at a speed of 4×10^{5}m/s if the magnetic field of the Earth is 1.5×10^{-5}T at that point and it is perpendicular to the velocity of the proton (as shown in the figure).Answer with magnitude and direction of the force.

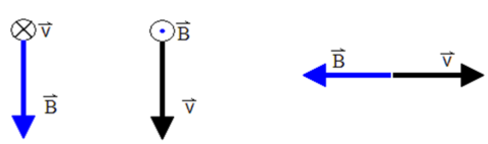


Solution: The force on a positive charge with the velocity and magnetic field vectors as shown in the figure can be found using the FBI rule. The force vector is shown in green:

The magnitude of the force is given by:

$F = Bqv\sin\theta = (1.5\times10^{-5}T)(1.6\times10^{-19}C)(4\times10^{5}m/s) =$ **9.6×10^{-19}N**

Problem 2.- Find the magnitude and indicate the direction of the force on a charge of 6.25 μC for each diagram shown, if B = 1.5 T and v = 29,979 m/s:



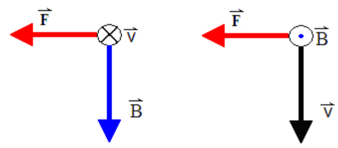
Solution: The magnitude of the force is given by

F=Bqv sinθ

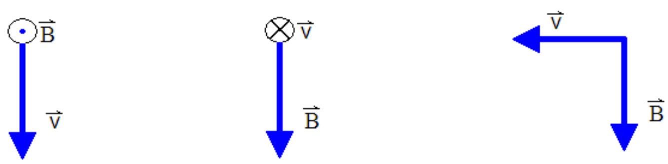
For cases (a) and (b) the force will be

F = (1.5 T)(6.25 μC)(29,979 m/s) = **0.28N**

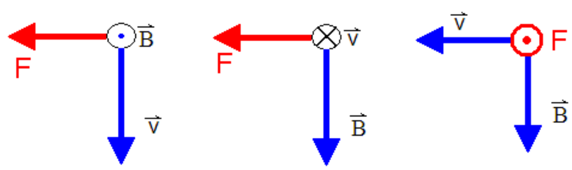
The force in case (c) is zero because the magnetic field and the velocity are collinear.
We find the direction of the force in cases (a) and (b) using the right hand rule.



Problem 2a.- Find the magnitude and direction of the force on a positive charge of 2.5 μC for each diagram shown, if B = 0.8 T and v = 1,200 m/s:



Solution: The magnitude is $F = Bqv\sin\angle_B^v = 0.8(2.5\times10^{-6})(1200)\sin 90^\circ =$ **0.0024 N**
The directions, found with the right-hand rule, are shown below.



Problem 3.- If two adjacent parallel wires carry electric current in the same direction, are they attracted to each other or repelled from each other? Give a short explanation.

Solution: They will attract each other. To prove this, consider one wire first and determine the magnetic field created at the position of the second wire and then use the right hand rule to determine the direction of the force on the second wire.

Problem 3a.- Calculate the magnitude of the force on one meter of wire due to another identical parallel wire. Take the currents to be 25A and the distance between the wires 0.8m

Solution: The magnitude of the force on one meter of wire due to the other is:

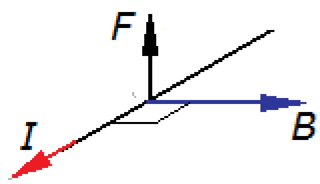
$$F = BIl\sin\angle_B^\ell = \frac{\mu_o I}{2\pi r} Il\sin\angle_B^\ell = \frac{4\pi\times10^{-7}(25)}{2\pi(0.8)}(25)\sin90^\circ = \mathbf{1.56\times10^{-4}\ N}$$

Problem 3b.- Determine the magnitude and direction of the force between two parallel wires 2.0 m long and 2.8cm apart, each carrying a current of 35A in opposite directions.

Solution: The direction of the force can be determined using the right-hand rule twice. Once to find the magnetic field produced by one wire on the other and the second time to find the force produced by this magnetic field. The correct answer is that the force is repulsive for currents that go in opposite directions.
The magnitude is given by:

$$F = \frac{\mu_o I_1}{2\pi r} L I_2 = \frac{4\pi\times10^{-7}\frac{\text{Tm}}{\text{A}}(35\text{A})}{2\pi(0.028\text{m})}(2.0\text{m})(35\text{A}) = \mathbf{0.0175\ N}$$

Problem 4.- A 2-meter-long wire has a mass of 0.0035 kg and is in a place where the magnetic field is horizontal, perpendicular to the wire and has a magnitude *B*=0.9 tesla. Calculate the current needed so the wire will levitate due to the magnetic force.



Solution: For the wire to float its weight will have to be equal to the magnetic force:

We use the equation $F = BI\ell\sin\angle_B^\ell$ Force on a current carrying wire in a magnetic field:

$$mg = BI\ell\sin\angle_B^\ell \rightarrow I = \frac{mg}{B\ell\sin\angle_B^\ell} = \frac{0.0035\times9.8}{0.9\times2\times\sin90^\circ} = \mathbf{0.019\ A}$$

Problem 4a.- How much current is necessary to produce a force of 0.55 N in a 3.8m long wire that is perpendicular to a magnetic field of 0.85T.

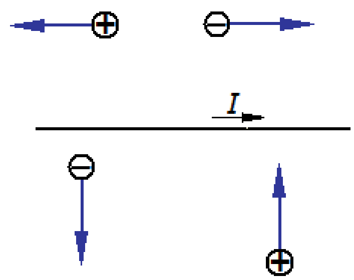
Solution: The force on a current carrying wire in a magnetic field is given by:

$$F = BLI\sin\theta$$

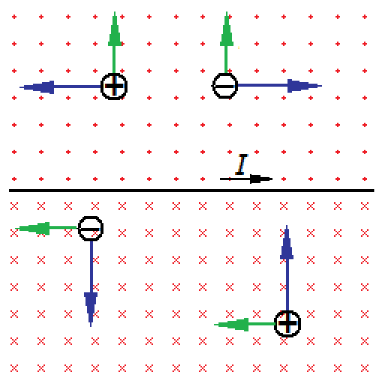
The problem states that the angle between the wire and the field is 90º, the field B is 0.85T, the length of the wire is L=3.8m, and the force is 0.55N, so we can solve for I as follows:

$$I = \frac{F}{BL\sin\theta} = \frac{0.55\text{N}}{(0.85\text{T})(3.8\text{m})\sin 90^{\circ}} = \mathbf{0.17\ A}$$

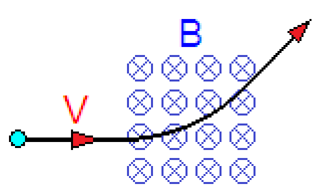
Problem 5.- The following charged particles are moving in the proximity of a current carrying wire. The sign of the charges is indicated (+ or -) as well as their velocity (with arrows).
For each charge, indicate the direction of the magnetic force due to the magnetic field produced by the wire.



Solution: The current generates a magnetic field given by the right-hand rule, which points in the directions indicated by the red crosses and dots.
Based on those directions we use the left-hand rule to find the forces on the charges (forces shown as green arrows)

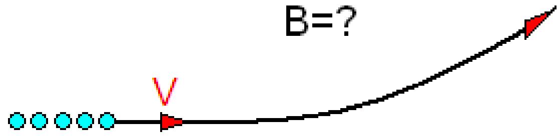


Problem 6.- The particle shown in the figure enters a region of magnetic field and is deflected upward. Is the charge of the particle positive or negative? Explain.

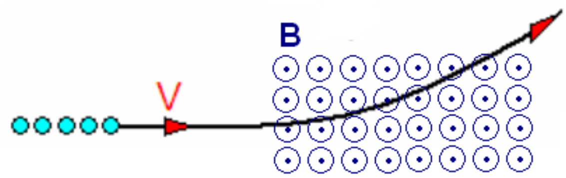


Solution: The right-hand rule indicates that the particle is **positive**.

Problem 6a.- You want to steer a beam of electrons (charge = -1.6×10^{-19}C) as shown in the figure. Indicate the direction of the magnetic field that will do this for you.



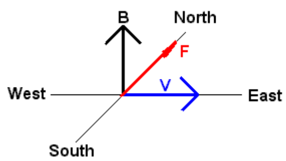
Solution: We use the FBI rule and invert the direction of the force because the charge is negative, so the field looks as follows:



Problem 7.- Determine the magnitude and direction of the force on an electron traveling at a speed of 5.75×10^{5} m/s horizontally to the east in the presence of a vertically upward magnetic field of 0.85T.

Charge of the electron $q_e = -1.6\times10^{-19}$ C

Solution: The "FBI" rule tells us the direction of the force, but notice that because the electron has negative charge, we have to reverse the direction given by the rule. As shown in the figure, the force will be due **North**.



The magnitude of the force is determined by:

$F = qvB\sin\theta = (1.6\times10^{-19}\text{C})(5.75\times10^{5}\text{ m/s})(0.85\text{T})(1) = \mathbf{7.82\times10^{-14}\ N}$

Problem 7a.- Find the magnitude and direction of the force on a positively charged cloud (charge = 1.5C) moving towards the East with a speed of 3m/s due to the magnetic field of the Earth that points North with a value of 45 μT.

Problem 8.- Describe the trajectory of an electron that enters a region of constant magnetic field at right angles (the angle between the field and the velocity is 90º).

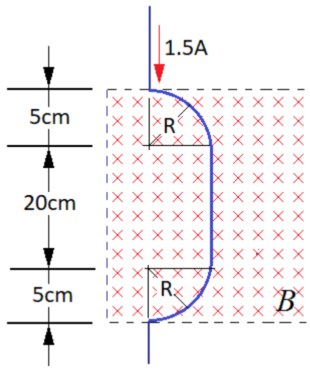
Solution: The trajectory of an electron (or any other charged particle) in a region of constant magnetic field entered at right angles is a **circle** (recall the example of the mass spectrometer).

Problem 9.- Determine the radius of the circular motion of a carbon-12 ion in a region of magnetic field B = 0.85T if its speed is 5.75×10^{6} m/s perpendicular to the magnetic field.
Charge of the carbon-12 ion = 1.6×10^{-19} C,
Mass of the ion = 2.0×10^{-26}kg

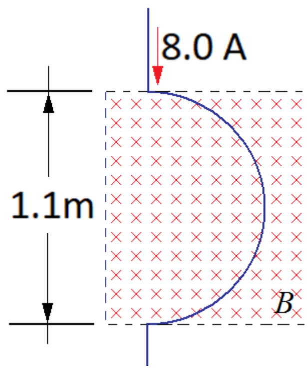
Solution: The magnetic force plays the role of the centripetal force, so:

$$F = \frac{mv^2}{R} = qvB \rightarrow R = \frac{mv}{qB} = \mathbf{0.85m}$$

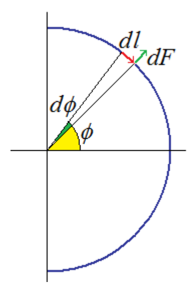
Problem 10.- Calculate the net force on the conductor, due to the magnetic field B = 0.25 T shown in the figure.



Problem 11.- Calculate the net force on the conductor, due to the magnetic field B = 0.25 T shown in the figure.



Solution: To calculate the force we divide the semicircle in small segments of arc *dl* as shown in the figure.



The differential of force is calculated with the equation:

$$dF = Idl \times B$$

But we notice that the field and the segment are perpendicular, so the magnitude of the vector differential is:

$dF = IBdl$

It would be incorrect to integrate this magnitude. This force is a radial vector as shown in the figure. Due to symmetry we only need to integrate the horizontal component:

$$dF_x = IB\cos\phi dl = IB\cos\phi R d\phi$$

$$F = \int_{-\pi/2}^{\pi/2} IB\cos\varphi R d\varphi = 2IBR = \mathbf{2.2N}$$

Problem 12.- Two transmission lines of length 75 m transport 150A of current in opposite directions and are separated 0,8 m. Calculate:

a) The magnetic field produced by one line at the position of the other.

b) The magnitude of the magnetic force between them. Is it repulsive or attractive?

c) Do you think this force should be considered when designing the structure to support the transmission lines?

d) During a short circuit, the current is 100 times the normal value. Is it important to consider the force in this case?

Solution:

a) Since the length of the wires is much more than the separation between them we can approximate them as infinite wires. Then:

$$B = \frac{\mu_\circ I}{2\pi r} = \frac{4\pi\times10^{-7}\times150}{2\pi\times0.8} = \mathbf{37.5\mu T}$$

b) To calculate the force, we use the equation

$$F = I\ell B = 150\times75\times37.5\times10^{-6} = \mathbf{0.422N}$$

Since the currents are opposite to each other, the force is repulsive.

c) A force of 0.422N is comparable to the weight of half an apple. It is not important for the design of the structures.

d) If the current is 100 times larger, the force will be 10,000 times larger. That is 4,220 N, which is the weight of 422kg. In this case it should be considered.

Problem 13.- Can you set a resting electron into motion with a magnetic field? Why?

Solution: The short answer is no, because if the electron is at rest the force due to the magnetic field is zero since F = qvB sinθ.

Note 1: It is true that it would be very difficult to have an electron at rest. A common misconception, for example, says that electrons will be at rest only at absolute zero temperature. That is not true. For example, in a metal there will be electrons moving at speeds of hundreds of thousands of meters per second even at absolute zero temperature. But the question is hypothetical.

Note 2: Besides charge, electrons have spin, which is intrinsic angular momentum. They have spin all the time, even if they are at rest. Also, associated with this spin, they have a magnetic moment. The magnetic moment can interact with a magnetic field. The main interaction is a torque that will cause the electron to "precess" around the magnetic field; a second order interaction is a force. However, for this force to exist, the magnetic field has to be inhomogeneous in space. So, a more precise answer would be yes, but only if the magnetic field is inhomogeneous in space.

Problem 14.- If a positively charged particle enters a region of uniform magnetic field which is perpendicular to the particle's velocity, will the kinetic energy of the particle increase, decrease or stay the same? Why?

Solution: Since the force due to the magnetic field is always perpendicular to the velocity, the work done on the particle is zero and so the kinetic energy is constant.

Problem 15.- Find the force acting on a particle that has a charge of 3.2×10^{-19}C if its velocity is given by $\vec{v} = (3\hat{i} + 4\hat{j} + 2\hat{k})\times10^{3} m/s$ and the magnetic field is $\vec{B} = 0.35\hat{k}\ tesla$

Solution: To find the force we use the cross product:

$$F = q\vec{v}\times\vec{B} = 3.2\times10^{-19}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3{,}000 & 4{,}000 & 2{,}000 \\ 0 & 0 & 0.35 \end{vmatrix} =$$

$$= 3.2\times10^{-19}\left\{\hat{i}\begin{vmatrix} 4{,}000 & 2{,}000 \\ 0 & 0.35 \end{vmatrix} - \hat{j}\begin{vmatrix} 3{,}000 & 2{,}000 \\ 0 & 0.35 \end{vmatrix} + \hat{k}\begin{vmatrix} 3{,}000 & 4{,}000 \\ 0 & 0 \end{vmatrix}\right\}$$

$$= 3.2\times10^{-19}\{1400\hat{i} - 1050\hat{j}\} = 4.48\times10^{-16}\hat{i} - 3.36\times10^{-16}\hat{j}$$

Problem 16.- A proton enters a region of magnetic field at a speed of 3×10^6 m/s. The field and the velocity are at right angles. Calculate the radius of the circle described by the trajectory of the particle if the magnetic field is 1.5×10^{-3}T.
Proton mass = 1.67×10^{-27} kg and charge q = 1.6×10^{-19}C