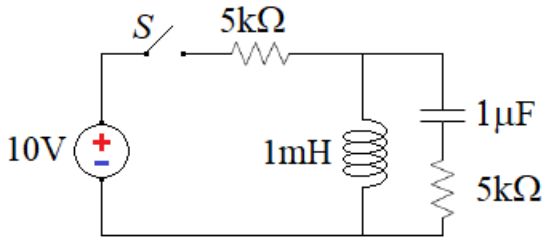
Physics II

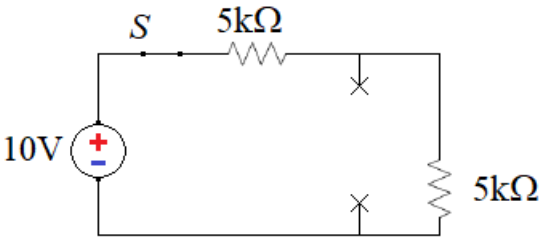
RLC circuits

Problem 1.- In the circuit shown below, the capacitor is initially uncharged and there is no current through the inductance. Calculate the current in the resistor that is in series with the source immediately after closing the switch S at $t = 0$ and a long time after that.



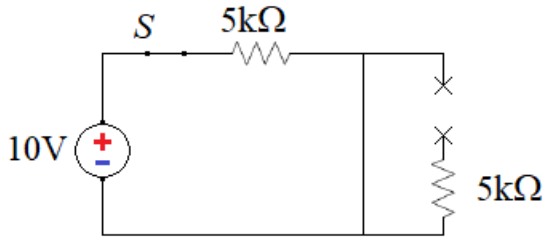
(A) $I(0) = 1\text{mA}; \quad I(\infty) = 2\text{mA}$
(B) $I(0) = 2\text{mA}; \quad I(\infty) = 1\text{mA}$
(C) $I(0) = 1\text{mA}; \quad I(\infty) = 0\text{mA}$
(D) $I(0) = 0\text{mA}; \quad I(\infty) = 1\text{mA}$
(E) $I(0) = 0\text{mA}; \quad I(\infty) = 2\text{mA}$

Solution: At $t = 0$ the capacitor behaves as a short-circuit because the voltage on its terminals will be zero (since its charge is zero). Meanwhile, the inductance will behave as an open circuit because its current is initially zero. The equivalent circuit is:



The initial current is: $I(0) = \dfrac{10\text{V}}{5\text{k}\Omega + 5\text{k}\Omega} = 1\text{mA}$

After a long time the capacitor will be completely charged and its current will be zero, so it will behave as an open circuit. This time the inductance will behave as a short-circuit since its current will be constant (and its voltage zero). The equivalent circuit will be:



The final current will be: $I(\infty) = \dfrac{10\text{V}}{5\text{k}\Omega} = 2\text{mA}$

Answer: (**A**)

Problem 2.- An RLC circuit is used to tune a radio to the frequency 103.7 MHz. The resistance of the circuit is 10 Ω and the inductance 2.0 μH. What is the best value of capacitor to be used?

(A) 200 pF (B) 50 pF (C) 1.2 pF (D) 0.2 pF (E) 0.02 pF

Solution: For the circuit to resonate at the given frequency we need:

$$\omega L = \frac{1}{\omega C} \rightarrow C = \frac{1}{\omega^2 L}$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (103.7\times10^6\,\text{Hz})^2 \times 2.0\times10^{-6}\,\text{H}} = \mathbf{1.18\ pF}$$

Answer: (**C**)

Problem 2a.- In an RLC circuit in series, the resistance is 330 ohm and the inductance 25 mH. The circuit is fed by a 20V amplitude generator with angular frequency 1,000 radians per second. What value of capacitor will give maximum current?

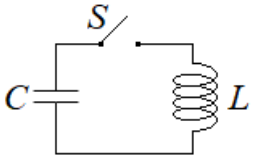
(A) 4nF (B) 40nF (C) 4μF (D) 40μF (E) 400μF

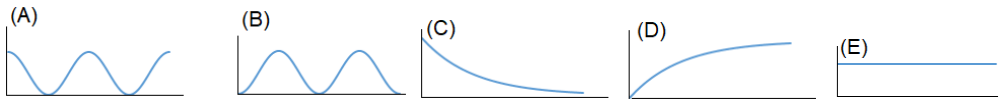
Solution: similar to the previous problem, we need resonance:

$$C = \frac{1}{\omega^2 L} = \frac{1}{(1000\text{rad/s})^2 \times 25\times10^{-3}\,\text{H}} = \mathbf{40\ \mu F}$$

Answer: (**D**)

Problem 3.- A circuit has a capacitor *C* and an inductor *L*. Initially the capacitor is charged and at time *t* = 0 the switch *S* is closed. Indicate which of the graphs better represents the energy stored in the inductor as a function of time. Assume they are ideal devices.





Solution: If the devices are ideal, there will be no losses and the energy will simply oscillate between the capacitor and the inductance. Since the initial current in the inductor is zero, its initial energy will be zero as well.

Answer: (**B**)