Physics II

Capacitor Circuits

$C = \dfrac{Q}{V}$ Definition of capacitance

$C = K\varepsilon_0 \dfrac{A}{d}$ Capacitance for parallel plates

$E = \dfrac{1}{2}CV^2$ Energy stored in a capacitor

Problem 1.- A 6μF capacitor is connected in series with a 12μF one. What will be the energy stored if we apply 5V to the circuit?

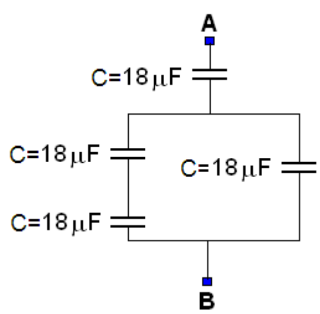
Solution: The capacitance is equivalent to

$$C = \frac{1}{\dfrac{1}{6\mu\text{F}} + \dfrac{1}{12\mu\text{F}}} = 4\mu\text{F}$$

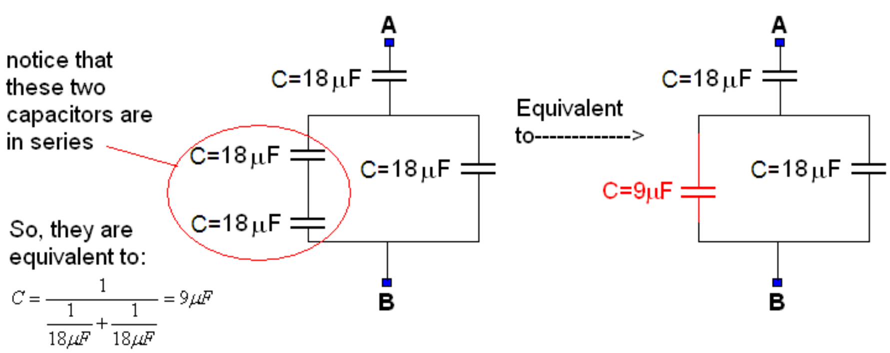
And the energy is

$$\frac{1}{2}CV^2 = \frac{1}{2}4\mu\text{F}(5\text{V})^2 = \mathbf{50\ \mu J}$$

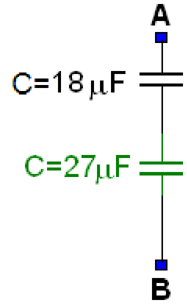
Problem 2.- Determine the capacitance of the following arrangement of capacitors and find how much energy is stored when you apply 10V between terminals A and B.



Solution:



Now, notice that the 9µF capacitor is in parallel with the 18µF one, so they together are equivalent to 18µF + 9 µF = 27 µF. Giving the circuit shown in the figure below:

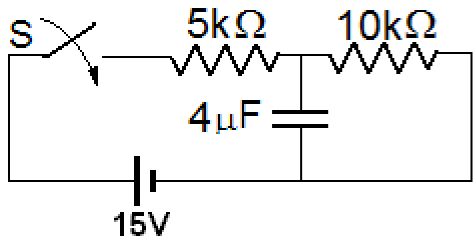


Finally, the equivalent of this last circuit is $C = \dfrac{1}{\dfrac{1}{18\mu F} + \dfrac{1}{27\mu F}} = \mathbf{10.8\mu F}$

And the energy stored is:

$$E = \frac{1}{2}CV^2 = \frac{1}{2}(10.8\mu F)(10V)^2 = \mathbf{0.54mJ}$$

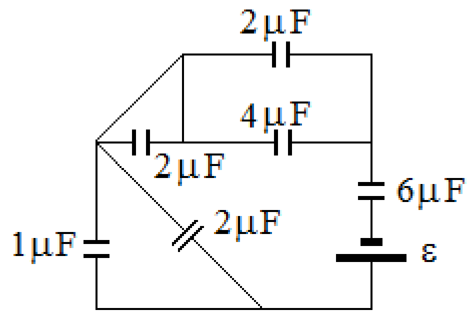
Problem 3.- Find the energy stored in the 4µF a long time after closing the switch S.



Solution: In DC circuits, capacitors behave like open circuits after a long time. The current in the resistors after a long time will be 15V/15kΩ = 1mA. The voltage in the 10kΩ resistor will then be 10V, which is also the voltage in the capacitor. Then the energy will be

$$E = \frac{1}{2}CV^2 = \frac{1}{2}(4\mu F)(10V)^2 = \mathbf{200\ \mu J}$$

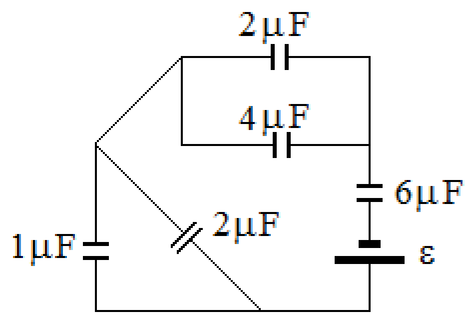
Problem 4.- In the circuit shown below, the voltage in the 6μF capacitor is 4V.



Find

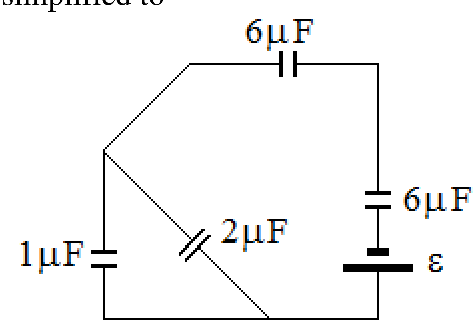
a) The capacitance of the whole circuit.
b) The source voltage.

Solution: We notice that the 2μF capacitor in the top left corner has both terminals connected to the same potential (they are joined by a wire), so it does not contribute to the circuit. We simplify it as follows.

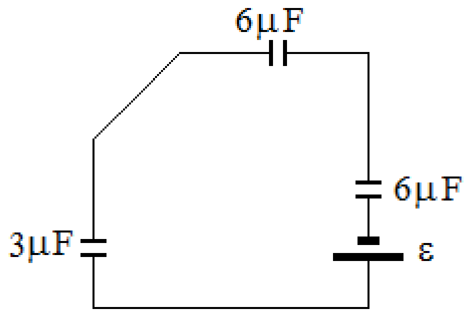


The charge in that capacitor is **zero**.

Next we notice that the 2μF and 4μF capacitors at the top are in parallel. They are equivalent to 6μF and the circuit is further simplified to

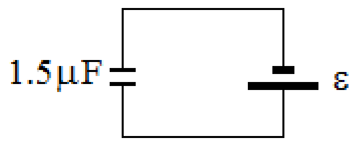


The 1μF and 2μF capacitors in the circuit are also in parallel as their terminals are connected to the same voltage. It does not matter that they do not appear physically in parallel as one is vertical and the other diagonal.
They are equivalent to 3μF.



Finally, the three capacitors in this last circuit are in series, so they are equivalent to

$$C = \frac{1}{\frac{1}{6\mu F} + \frac{1}{6\mu F} + \frac{1}{3\mu F}} = \mathbf{1.5\mu F}$$



Since the voltage in the 6μF capacitor is 4V its charge is

$$Q = CV = (6\mu F)(4V) = 24\mu C$$

This is the same charge of the equivalent circuit. Recall that when capacitors are in series they have the same charge. Then

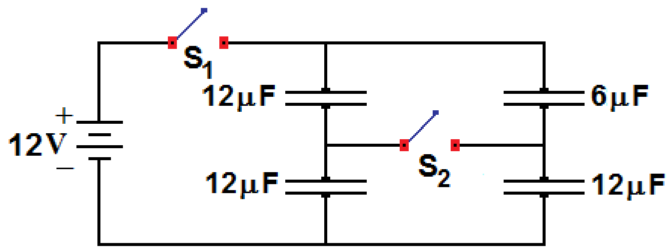
Q = 24μF

With the value of the charge and the equivalent capacitance we can calculate the source voltage.

$$V = \frac{Q}{C} = \frac{24\mu C}{1.5\mu F} = \mathbf{16V}$$

Problem 5.- Consider the circuit shown in the figure.

a) With S_1 closed and S_2 open, calculate the total capacitance.
b) In that same position, calculate the total energy stored in the circuit.
c) With both S_1 and S_2 closed, calculate the total capacitance.
d) In this last position, calculate the total stored energy.



Solution:

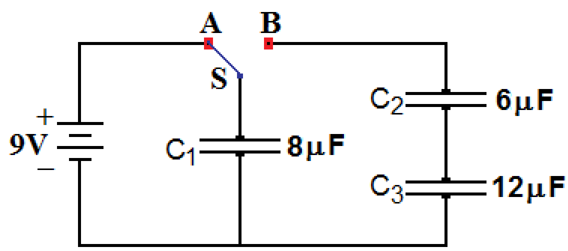
a) $C = \dfrac{1}{\dfrac{1}{12}+\dfrac{1}{12}} + \dfrac{1}{\dfrac{1}{6}+\dfrac{1}{12}} = 6+4 = \mathbf{12\mu F}$

b) $E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}12\mu\text{F}\times(12\text{V})^2 = \mathbf{864\mu J}$

c) $C = \dfrac{1}{\dfrac{1}{12+6}+\dfrac{1}{12+12}} = \dfrac{1}{\dfrac{1}{18}+\dfrac{1}{24}} = \mathbf{10.3\mu F}$

d) $E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}10.3\times(12)^2 = \mathbf{741\mu J}$

Problem 6.- In the circuit shown, S is first in A and C_2 and C_3 are initially uncharged. Then, S is switched to B, connecting C_1 with C_2 and C_3.



Answer:
a) What is the charge in C_1 when S is in A?
b) What is the charge in C_1 long after changing S to B?
c) What is the final voltage in C_1?
d) What is the final voltage in C_2?
e) What is the final voltage in C_3?

Solution:

a) $Q_1 = CV = 9\text{V} \times 8\mu\text{F} = 72\mu\text{C}$

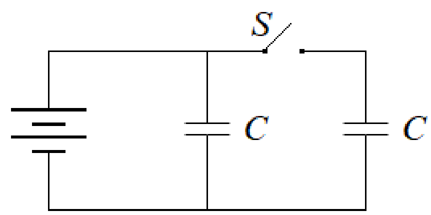
b) $V = \dfrac{Q}{C} = \dfrac{72\mu\text{C}}{12\mu\text{F}} = 6\text{V}$

c) $Q_1 = CV = 6\text{V} \times 8\mu\text{F} = 48\mu\text{C}$

d) $V_2 = \dfrac{Q}{C} = \dfrac{72\mu\text{C} - 48\mu\text{C}}{6\mu\text{F}} = 4\text{V}$

e) $V_3 = 6\text{V} - 4\text{V} = 2\text{V}$

Problem 7.- the figure shows two identical capacitors. Initially, with the switch S open, one capacitor is uncharged and the other has a charge Q_0. The stored energy in this last capacitor is U_0. The switch is closed and after a while the capacitors have charges Q_1 and Q_2. The voltage and charge in each capacitor are V_1, U_1, V_2 y U_2. Which of the following alternatives is incorrect?



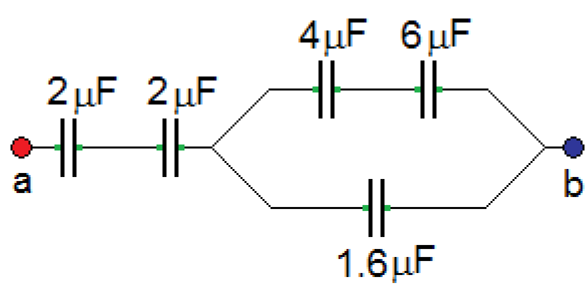
(A) $Q_0 = \dfrac{Q_1 + Q_2}{2}$ (B) $Q_1 = Q_2$ (C) $V_1 = V_2$ (D) $U_1 = U_2$ (E) $U_0 = U_1 + U_2$

Problem 8.-

i) Calculate the equivalent capacitance of the circuit from the terminals a and b.

ii) Calculate the charge in each capacitor if we connect a 16V source across a and b.

iii) Calculate the voltage across the 4μF capacitor in case (ii).



Solution: To solve the problem, we can simplify the circuit and later we go back over our steps deducing voltages and charges in each capacitor. We use the equation:

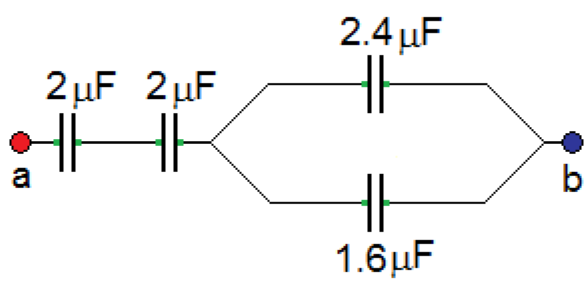
$$C = \frac{Q}{V}$$

Besides, we know that capacitors in series have the same charge and capacitors in parallel have the same voltage. To calculate equivalent capacitances we add capacitances in parallel and take the inverse of the sum of the inverses if in series.

In the original circuit we notice that the 4μF and 6μF capacitors are in series, which give us an equivalent:

$$C = \frac{1}{\frac{1}{4\mu\text{F}} + \frac{1}{6\mu\text{F}}} = 2.4\mu\text{F}$$

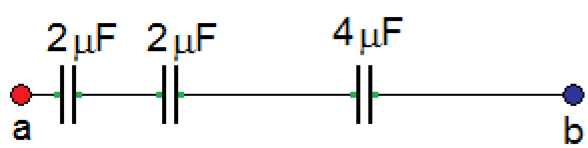
We obtain the circuit shown below:



This 2.4μF is in parallel with a 1.6μF capacitor. They are equivalent to:

$$C = 2.4\mu\text{F} + 1.6\mu\text{F} = 4\mu\text{F}$$

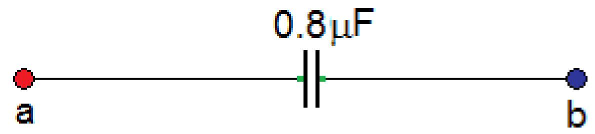
This gives us the circuit below:



These three capacitors are in series and they are equivalent to:

$$C = \frac{1}{\frac{1}{2\mu\text{F}} + \frac{1}{2\mu\text{F}} + \frac{1}{4\mu\text{F}}} = 0.8\mu\text{F}$$

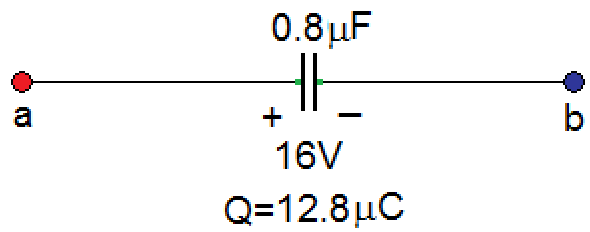
The circuit equivalent reduces to:



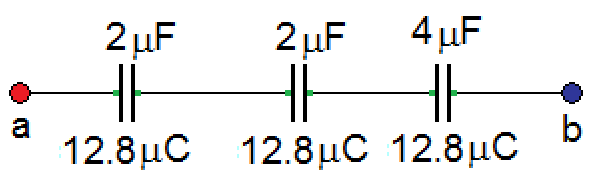
We can answer the first question: (a) **0.8 μF**

Now we go back. We notice that in the last circuit there are 16V applied between terminals a and b. We can calculate the charge stored in the equivalent capacitor:

$Q = CV = 16\text{V} \times 0.8\mu\text{F} = 12.8\mu\text{C}$



We take a step back to the previous circuit and notice that the capacitors will have the same charge because they are in series:

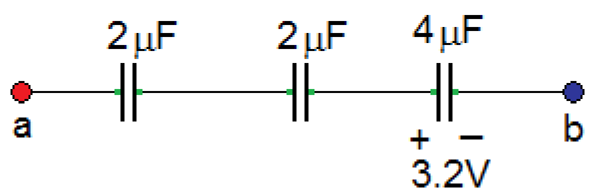


There are **12.8μC** stored in the 2μF capacitors.

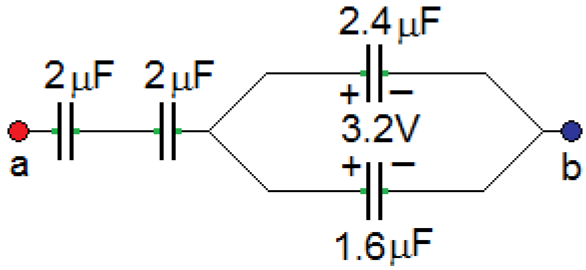
We can calculate the voltage in the 4μF capacitor:

$$V = \frac{Q}{C} = \frac{12.8\mu\text{C}}{4\mu\text{F}} = 3.2\text{V}$$

The equivalent circuit becomes this



This voltage is the same for the two capacitors that are in parallel as shown below:

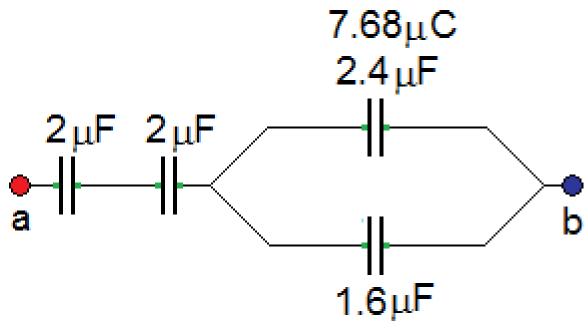


The charge stored in the 1.6μF capacitor is

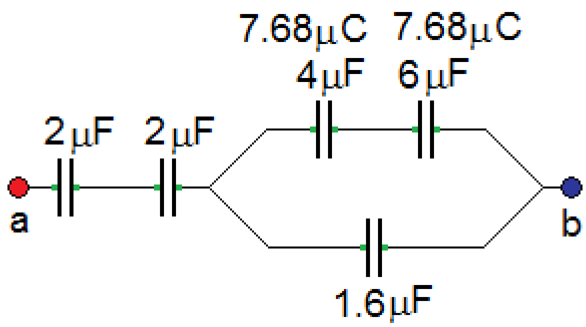
$Q = CV = 3.2\text{V} \times 1.6\mu\text{F} = \mathbf{5.12\mu C}$

The charged stored in the 2.4μF equivalent capacitor is

$Q = CV = 3.2\text{V} \times 2.4\mu\text{F} = 7.68\mu\text{C}$



This charge is the same for the two capacitors in series that make up the equivalent. So going back one step to the original circuit we get:



The voltage across the 4μF capacitor is

$$V = \frac{Q}{C} = \frac{7.68\mu\text{C}}{4\mu\text{F}} = \mathbf{1.92V}$$