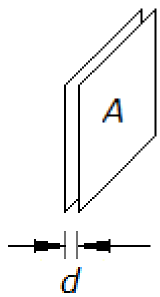
Physics II

Capacitors and dielectrics

$C = \frac{Q}{V}$	Definition of capacitance
$C = K\varepsilon_o \frac{A}{d}$	Capacitance for parallel plates
$E_{stored} = \frac{1}{2}CV^2$	Energy stored in a capacitor

Problem 1.- To make a 0.47μF capacitor, what area must the plates have if the air gap between them is d=0.25mm?



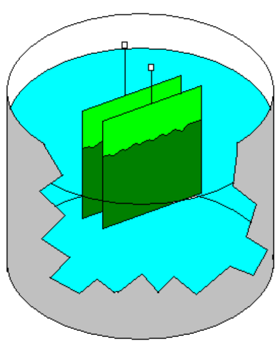
Solution: We can find the required area using the equation for a flat capacitor:

$$C = \varepsilon_o \frac{A}{d} \rightarrow A = \frac{Cd}{\varepsilon_o} = \frac{0.47\times10^{-6}\times0.25\times10^{-3}}{8.85\times10^{-12}} = \mathbf{13.3\ m^2}$$

Problem 2.- Calculate the capacitance of the two plates shown in the figure when they are completely submerged in a liquid whose dielectric constant (K) is 3.4

Area of the plates = 0.24 m^2, distance between the plates = 0.36 mm.

Dielectric permittivity of vacuum, $\varepsilon_o = 8.85\times10^{-12}\ F/m$



Solution: The equation for the capacitance of a pair of parallel plates is

$$C = \varepsilon_o \frac{A}{d}$$

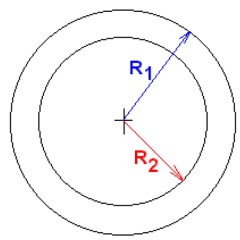
This is valid in vacuum and very approximately, also in air. In this liquid it will have to be corrected by multiplying by the dielectric constant, so:

$$C = K\varepsilon_o \frac{A}{d} = 3.4\times(8.85\times10^{-12}\,F/m)\frac{0.24m^2}{0.00036m} = \mathbf{20.1\ nF}$$

Problem 2a.- If it weren't for practical reasons that prevent it, water would be a good choice to make capacitors since its dielectric constant is very large (K_{water}=81). Calculate the capacitance (in farads) of two plates submerged in water if their area is 0.25 m^2 and the distance between them is 0.55 mm.

$\varepsilon_o = 8.85\times10^{-12}\,F/m$

Problem 3.- Calculate the capacitance of two concentric spheres of radius R_1=2.0m and R_2=1.5m.



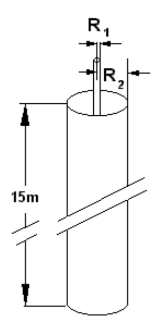
Solution: By definition, the capacitance is the charge divided by the electric potential. Consider the internal sphere to have a charge Q and the external sphere -Q. Then the potential difference is:

$$V = k\frac{Q}{R_2} - k\frac{Q}{R_1}$$

And the capacitance is: $C = \frac{Q}{V} = \frac{Q}{k\frac{Q}{R_2} - k\frac{Q}{R_1}} = \frac{1}{9\times10^9\left(\frac{1}{1.5} - \frac{1}{2}\right)} = \mathbf{667\ pF}$

Problem 4.- A wave guide consists of two concentric conductors as shown schematically in the figure below. Calculate the capacitance between the conductors if the length of the guide is 15m and the radii are R_1 = 1mm and R_2 = 20.1mm

Suggestion: Assume the interior conductor has a charge Q. Use Gauss's law to find the electric field in the gap and integrate to find the voltage. Then use the definition of C.



Solution: Assuming the interior conductor has a charge Q then the electric field in the gap can be calculated using Gauss's law

$$E(2\pi rL) = 4\pi kQ \rightarrow E = \frac{2kQ}{rL}$$

The electric potential is $V = \int_{R_1}^{R_2} \frac{2kQ}{rL} dr = \frac{2kQ}{L} \ln\left(\frac{R_2}{R_1}\right)$

And the capacitance is:

$$C = \frac{Q}{V} = \frac{Q}{\frac{2kQ}{L}\ln\left(\frac{R_2}{R_1}\right)} = \frac{L}{2k\ln\left(\frac{R_2}{R_1}\right)} = \frac{15}{2\times(9\times10^9)\ln\left(\frac{20.1}{1}\right)} = \mathbf{277\ pF}$$

Problem 5.- Some people think that we could use capacitors to store energy in case of blackout. Imagine that you get your hands on a humongous capacitor of 2 farads. What would be the voltage across this device if you want to store 1kwh of energy?

1 kwh = 3.6×10^6 joules

Solution: $3.6\times10^6 = \frac{1}{2}2V^2 \rightarrow V =$ **1,900 volts**

Problem 6.- A cardiac defibrillator is used to shock a heart that is beating erratically. A capacitor in this device is charged to 4.5 kV and stores 1200 J of energy. What is the capacitance?

Solution: Since the energy stored in the capacitor is

$$\text{Energy} = \frac{1}{2}CV^2$$

The capacitance is: $C = \frac{2\times\text{Energy}}{V^2} = \frac{2\times1200J}{(4500V)^2} = \mathbf{118\ \mu F}$

Problem 7.- A home-made capacitor consists of two sheets of aluminum foil of area $0.25m^2$ separated by a 0.12mm-thick paper (dielectric constant=3.2). How much energy will be stored in this capacitor if you connect it to a source of 110 volts?

Solution: The capacitance is given by:

$$C = K\varepsilon_o\frac{A}{d} = (3.2)(8.85\times10^{-12}F/m)\frac{0.25m^2}{0.00012m} = \mathbf{59nF}$$

And the energy stored is: $E = \frac{1}{2}CV^2 = 59nF(110V)^2 = \mathbf{357\mu J}$

Problem 8.- A 6µF capacitor is connected in series with a 12µF one. What will be the energy stored if we apply 5V to the circuit?

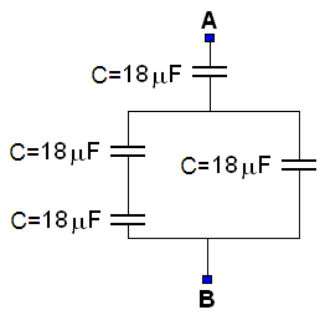
Solution: The capacitance is equivalent to

$$C = \frac{1}{\frac{1}{6\mu F} + \frac{1}{12\mu F}} = 4\mu F$$

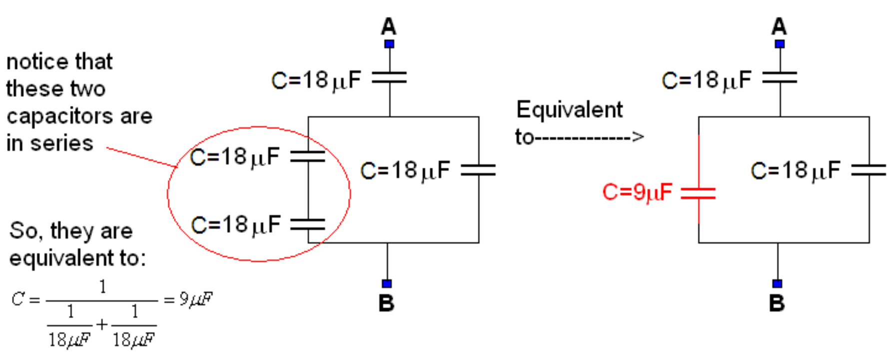
And the energy is

$$\frac{1}{2}CV^2 = \frac{1}{2}4\mu F(5V)^2 = \mathbf{50\ \mu J}$$

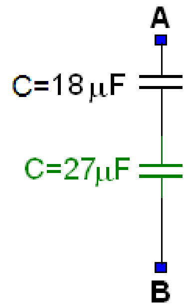
Problem 9.- Determine the capacitance of the following arrangement of capacitors and find how much energy is stored when you apply 10V between terminals A and B.



Solution:



Now, notice that the 9µF capacitor is in parallel with the 18µF one, so they together are equivalent to 18µF + 9 µF = 27 µF. Giving the circuit shown in the figure below:

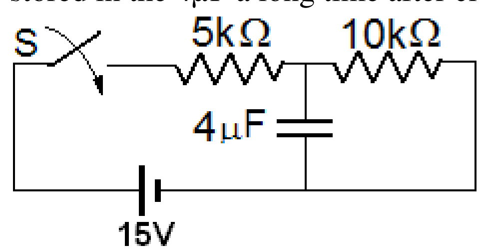


Finally, the equivalent of this last circuit is $C = \dfrac{1}{\dfrac{1}{18\mu F} + \dfrac{1}{27\mu F}} =$ **10.8μF**

And the energy stored is:

$$E = \frac{1}{2}CV^2 = \frac{1}{2}(10.8\mu F)(10V)^2 = \mathbf{0.54mJ}$$

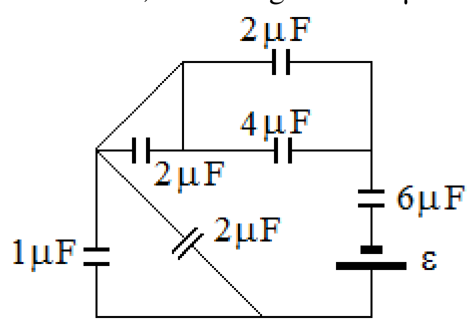
Problem 10.- Find the energy stored in the 4μF a long time after closing the switch S.



Solution: In DC circuits capacitors behave like open circuits after a long time. The current in the resistors after a long time will be 15V/15kΩ = 1mA. The voltage in the 10kΩ resistor will then be 10V, which is also the voltage in the capacitor. Then the energy will be

$$E = \frac{1}{2}CV^2 = \frac{1}{2}(4\mu F)(10V)^2 = \mathbf{200\ \mu J}$$

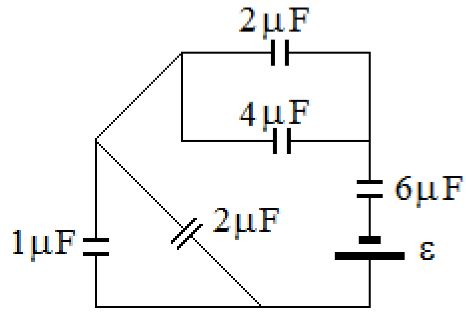
Problem 11.- In the circuit shown below, the voltage in the 6μF is 4V.



Find

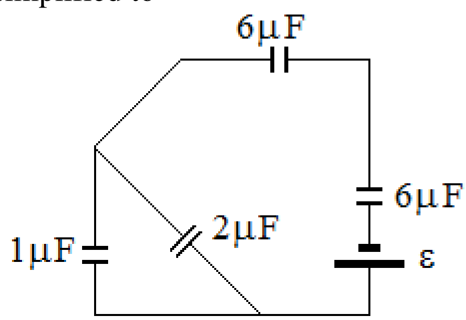
a) The capacitance of the whole circuit.
b) The source voltage.

Solution: We notice that the 2μF capacitor in the top left corner has both terminals connected to the same potential (they are joined by a wire), so it does not contribute to the circuit. We simplify it as follows.

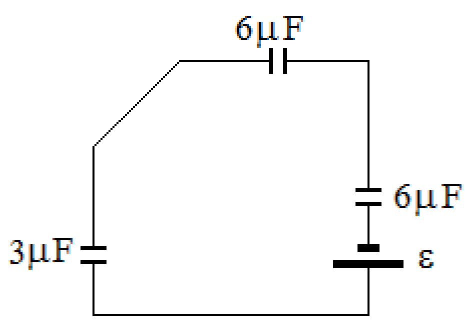


The charge in that capacitor is **zero**.

Next we notice that the 2μF and 4μF capacitors at the top are in parallel. They are equivalent to 6μF and the circuit is further simplified to

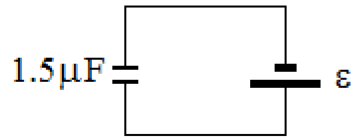


The 1μF and 2μF capacitors in the circuit are also in parallel as their terminals are connected to the same voltage. It does not matter that they do not appear physically in parallel as one is vertical and the other diagonal.
They are equivalent to 3 μF.



Finally, the three capacitors in this last circuit are in series, so they are equivalent to

$$C = \frac{1}{\frac{1}{6\mu F} + \frac{1}{6\mu F} + \frac{1}{3\mu F}} = \mathbf{1.5\mu F}$$



Since the voltage in the 6μF capacitor is 4V its charge is

$$Q = CV = (6\mu F)(4V) = 24\mu C$$

This is the same charge of the equivalent circuit. Recall that when capacitors are in series they have the same charge. Then

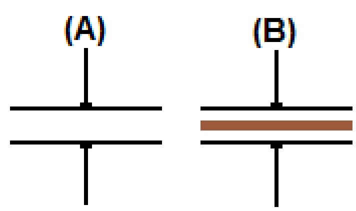
Q = 24μF

With the value of the charge and the equivalent capacitance we can calculate the source voltage.

$$V = \frac{Q}{C} = \frac{24\mu C}{1.5\mu F} = \mathbf{16V}$$

Problem 12.- We have a parallel plates capacitor with area A_o which are separated a distance d_o as shown in (A). In that case, its capacitance is 12nF and it is initially charged with Q = 144nC. Next, a conducting plate with thickness $d_o/2$ is inserted in the gap as shown in (B) without disturbing the original charge of 144nC. Answer

a) What is the initial voltage in the capacitor?
b) What is the initial energy in the capacitor?
c) What is the new capacitance, after inserting the additional plate?
d) What is the energy stored after inserting the additional plate?
e) When inserting the plate, is it attracted or repelled by the capacitor?



Solution:

a) $V = \frac{Q}{C} = \frac{144nF}{12nF} = \mathbf{6V}$

b) $E = \frac{1}{2}CV^2 = \frac{1}{2}12nF \times (6V)^2 = \mathbf{216nJ}$

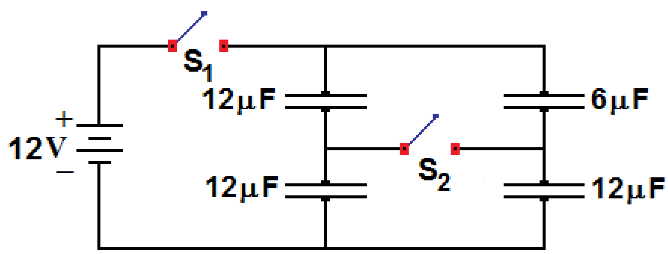
c) $C = \frac{1}{\frac{1}{48nF} + \frac{1}{48nF}} = \mathbf{24nF}$

d) $E = \frac{Q^2}{2C} = \frac{(144nC)^2}{2\times 24nF}$ = **108nJ**

e) Since the final stored energy is less than the initial the external work is negative. **The capacitor attracts the plate.**

Problem 13.- Consider the circuit shown in the figure.

a) With S_1 closed and S_2 open, calculate the total capacitance.
b) In that same position, calculate the total energy stored in the circuit.
c) With both S_1 and S_2 closed, calculate the total capacitance.
d) In this last position, calculate the total stored energy.



Solution:

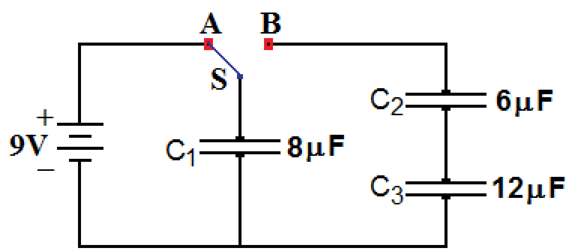
a) $C = \frac{1}{\frac{1}{12}+\frac{1}{12}} + \frac{1}{\frac{1}{6}+\frac{1}{12}} = 6+4 =$ **12μF**

b) $E = \frac{1}{2}CV^2 = \frac{1}{2}12\mu F\times(12V)^2$ = **864μJ**

c) $C = \frac{1}{\frac{1}{12+6}+\frac{1}{12+12}} = \frac{1}{\frac{1}{18}+\frac{1}{24}}$ = **10.3μF**

d) $E = \frac{1}{2}CV^2 = \frac{1}{2}10.3\times(12)^2$ = **741μJ**

Problem 14.- In the circuit shown, S is first in A and C_2 and C_3 are initially uncharged. Then, S is switched to B, connecting C_1 with C_2 and C_3.

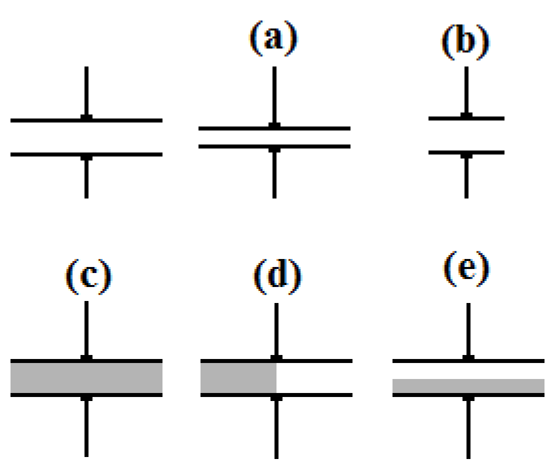


Answer:
a) What is the charge in C_1 when S is in A?
b) What is the charge in C_1 long after changing S to B?
c) What is the final voltage in C_1?
d) What is the final voltage in C_2?
e) What is the final voltage in C_3?

Solution:
a) $Q_1 = CV = 9V \times 8\mu F = 72\mu C$
b) $V = \frac{Q}{C} = \frac{72\mu C}{12\mu F} = 6V$
c) $Q_1 = CV = 6V \times 8\mu F = 48\mu C$
d) $V_2 = \frac{Q}{C} = \frac{72\mu C - 48\mu C}{6\mu F} = 4V$
e) $V_3 = 6V - 4V = 2V$

Problem 15.- In the first figure we see a parallel plates capacitor with area A_o separated in vacuum a distance d_o, whose capacity is $C_o = 12\mu F$.
Answer:

a) What will be the capacitance if you reduce the distance between the plates to $d_o/2$?
b) What will be the capacitance if you reduce the area to $A_o/2$?
c) What will be the capacitance if you fill the space between the plates with a dielectric with constant K = 2 as shown in (c)?
d) What will be the capacitance if you fill half the space between the plates with a dielectric with constant K = 2 as shown in (d)?
e) What will be the capacitance if you fill half the space between the plates with a dielectric with constant K = 2 as shown in (e)?



Solution:

a) $C = \frac{C_o}{1/2} =$ **24μF**

b) $C = \frac{C_o}{2} =$ **6μF**

c) $C = 2C_o =$ **24μF**

d) $C = 2\frac{C_o}{2} + \frac{C_o}{2} =$ **18μF**

e) $C = \frac{1}{\frac{1}{4C_o} + \frac{1}{2C_o}} = \frac{4}{3}C_o =$ **16μF**