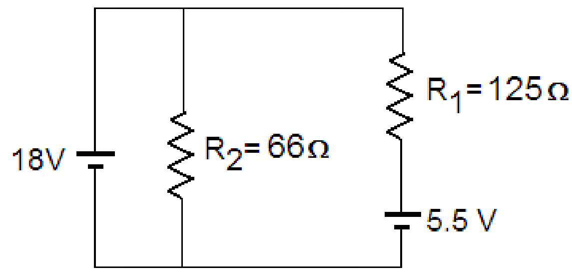
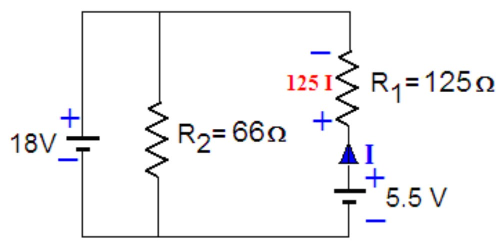
Physics II

Kirchhoff

Problem 1.- Determine the magnitude and direction (up or down) of the current through R_1.



Solution: A voltage equation for the outer loop of the circuit looks like this:

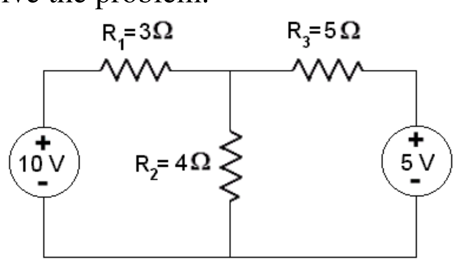


The equation is

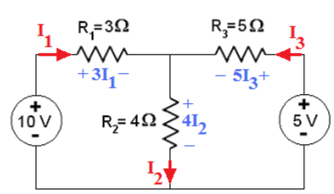
$18V+(125\Omega)I-5.5V=0$ assuming the current direction is up.

Solving for I we get **-0.1A**, where the minus sign indicates that the direction is **down**.

Problem 2.- We want to find the current through the resistors R_1, R_2 and R_3. Write down the equations that you need and solve the problem.



Solution: If we assume the currents are in the direction shown in the figure (red arrows) the drop in voltage across the resistors are indicated in blue.



The equations that we need are:

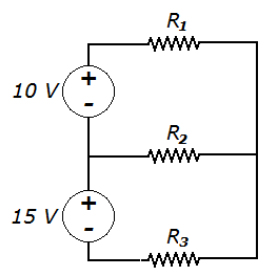
$$I_1 + I_3 = I_2$$
$$10 - 3I_1 - 4I_2 = 0$$
$$5 - 5I_3 - 4I_2 = 0$$

Which we solve, finding:

I$_1$ = **1.49 A**
I$_2$ = **1.38 A**
I$_3$ = **-0.11 A**

Here again, the negative sign of I$_3$ indicates that it is in the opposite direction as initially assumed.

Problem 3.- Write down the equations to find the currents in the resistors. The values of the resistances are R_1=10Ω, R_2=15Ω and R_3=12Ω.



Solution: We could define three currents going to the right I_1, I_2 and I_3.

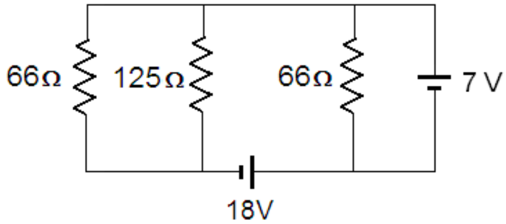
Then the first equation is: $I_1 + I_2 + I_3 = 0$

Then we need two loop equations, for example:

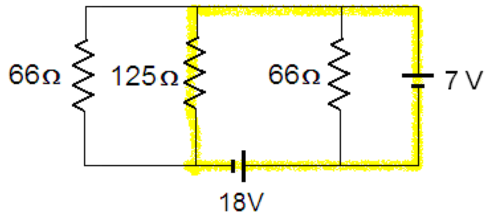
$15V - R_2 I_2 + R_3 I_3 = 0$ and

$10V - R_1 I_1 + R_2 I_2 = 0$

Problem 4.- Find the current through the 125Ω resistor.

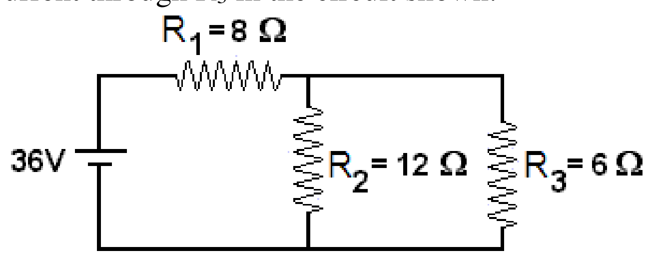


Solution: Following the loop highlighted in the figure we apply Kirchhoff's rules:



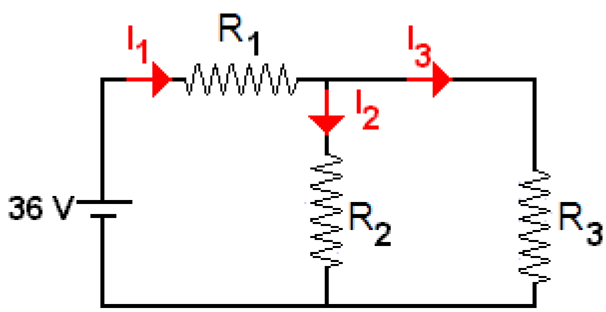
$$125I - 7 - 18 = 0 \rightarrow I = \frac{7+18}{125} = \mathbf{0.2\ A}$$

Problem 5.- Find the current through R_3 in the circuit shown:



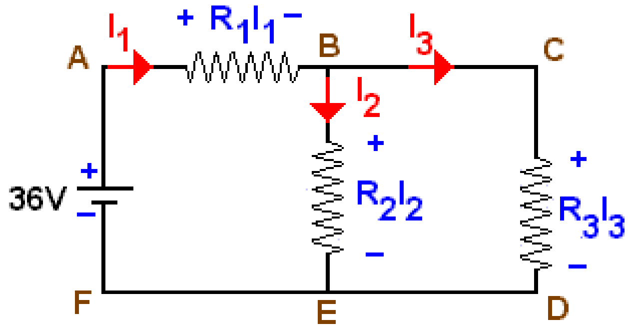
Solution: There are several ways to solve this problem. Let us look at two of them:

(i) First approach. Let us write the voltage and current equations using Kirchhoff's rules. To do this we first define three currents in our circuit:



With these currents, the voltage on each resistor will be given by Ohm's law, $V_1 = R_1I_1$, $V_2 = R_2I_2$, and $V_3 = R_3I_3$.

Recall that the current in a resistor goes from positive to negative, from higher potential to lower potential, so the circuit looks like this:



Now we are ready to write the equations.

Let us look first at the left loop that goes from the source to the resistor R_1, resistor R_2 and back to the source (loop ABEFA)

$-R_1I_1 - R_2I_2 + 36V = 0$ (Equation 1)

We can write another equation for the voltages in the loop ABCDEFA as follows:

$-R_1I_1 - R_3I_3 + 36V = 0$ (Equation 2)

There is another loop, BCDEB, but we do not gain anything by writing its equation, since it would not be an independent equation. Of course, you can write its equation and use it *instead* of equation 1 or 2 if you want.

We can write an additional equation for node "B". According to Kirchhoff's rule for current, the net current going to a node should be zero. I_1 goes to the node B so we write it with a positive sign. I_2 and I_3 go away from the node, so we write them with a minus sign in the equation:

$I_1 - I_2 - I_3 = 0$ (Equation 3)

We now have three equations and three variables, so the problem is solved from the point of view of physics.

To find I_3 we could use any standard method for solving simultaneous equations, for example using determinants, we write the three equations in matrix form:

$$\begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 36 \\ 36 \\ 0 \end{bmatrix}$$

The determinant of the system is:

$$\det\left(\begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix}\right) = R_1 \det\left(\begin{bmatrix} 0 & R_3 \\ -1 & -1 \end{bmatrix}\right) - R_2 \det\left(\begin{bmatrix} R_1 & R_3 \\ 1 & -1 \end{bmatrix}\right) = R_1R_3 + R_2R_1 + R_2R_3$$

And the determinant of I_3 is:

$$\det\left(\begin{bmatrix} R_1 & R_2 & 36 \\ R_1 & 0 & 36 \\ 1 & -1 & 0 \end{bmatrix}\right) = R_1 \det\left(\begin{bmatrix} 0 & 36 \\ -1 & 0 \end{bmatrix}\right) - R_2 \det\left(\begin{bmatrix} R_1 & 36 \\ 1 & 0 \end{bmatrix}\right) + 36 \det\left(\begin{bmatrix} R_1 & 0 \\ 1 & -1 \end{bmatrix}\right) = 36R_2$$

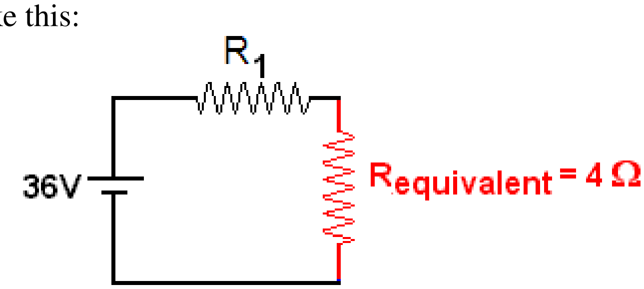
So I_3 is:

$$I_3 = \frac{36R_2}{R_1R_3 + R_2R_1 + R_2R_3} = \frac{36(12)}{(8)(6) + (12)(8) + (12)(6)} = \mathbf{2.0\ A}$$

(ii) Second approach. A somewhat simpler approach would be the following:
Consider resistors R_2 and R_3, they are in parallel because they share the same voltage. So we can replace them by a single resistor with a value of:

$$R_{\text{equivalent}} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{6\Omega} + \frac{1}{12\Omega}} = 4\Omega$$

So the circuit looks like this:



The total current through the circuit is then $I_{\text{Total}} = \dfrac{36}{R_1 + 4}$

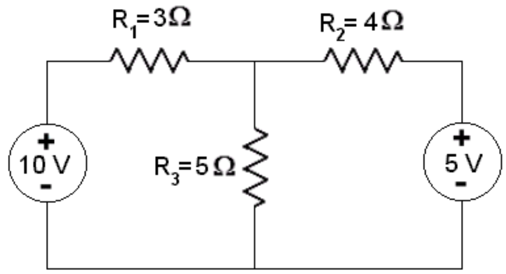
The voltage drop in the resistor R_1 is then: $V_{R1} = I_{\text{Total}} R_1 = \dfrac{R_1 36}{R_1 + 4}$

So, the voltage across the equivalent resistor is $36 - V_{R1} = 36 - \dfrac{R_1 36}{R_1 + 4}$

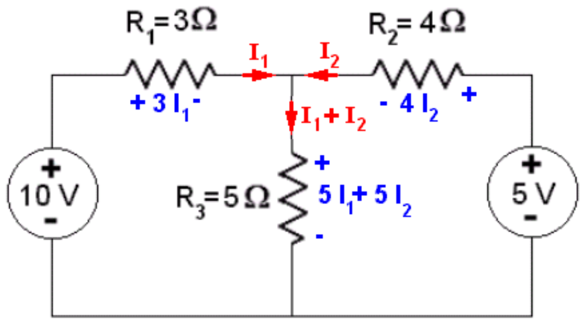
Finally, the current through R_3 is this voltage divided by R_3:

$$I_3 = \frac{36 - \frac{R_1 36}{R_1 + 4}}{R_3} = \frac{36 - \frac{(8)(36)}{8+4}}{6} = \frac{36 - 24}{6} = \mathbf{2.0\ A}$$

Problem 6.- Find the current through the resistors R_1, R_2 and R_3.



Solution: Let's assume that the currents are as shown:



The loop equations are:

$$10 - 3I_1 - (5I_1 + 5I_2) = 0$$
$$-5 + 4I_2 + (5I_1 + 5I_2) = 0$$

And writing them in the standard form:

$$-8I_1 - 5I_2 = -10$$

$$5I_1 + 9I_2 = 5$$

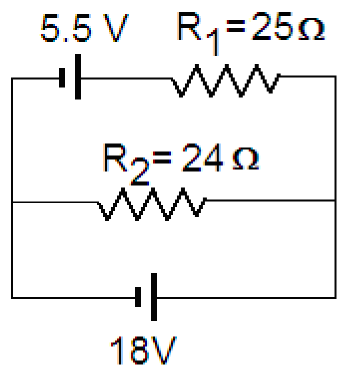
The solutions are:

$$I_1 = \frac{\begin{vmatrix} -10 & -5 \\ 5 & 9 \end{vmatrix}}{\begin{vmatrix} -8 & -5 \\ 5 & 9 \end{vmatrix}} = \frac{-90+25}{-72+25} = \frac{-65}{-47} = \mathbf{1.38\ A}$$

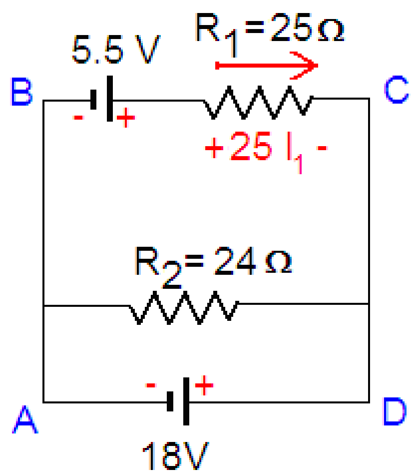
$$I_2 = \frac{\begin{vmatrix} -8 & -10 \\ 5 & 5 \end{vmatrix}}{\begin{vmatrix} -8 & -5 \\ 5 & 9 \end{vmatrix}} = \frac{-40+50}{-72+25} = \frac{10}{-47} = \mathbf{-0.21\ A}$$

$$I_3 = I_1 + I_2 = \mathbf{1.17\ A}$$

Problem 7.- Determine the magnitude and direction (left or right) of the current through R_1.



Solution: We concentrate only on the external loop (ABCD) because that makes it easier to find the answer here. If we assume the current goes from left to right through resistor R_1 the circuit looks like this:



Recall that when writing Kirchhoff's equations, we should use the correct sign. A good practical rule is that if we come out through the "+" sign when going around the loop we should use a plus in the equation and if it's a "-" sign, then we should use a minus in the equation as well.

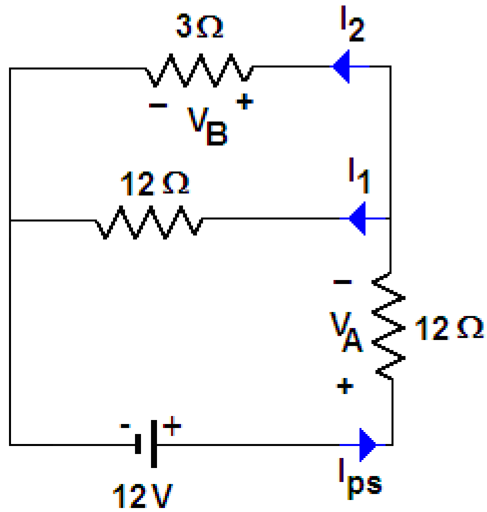
The equation is:

$$5.5 - 25I_1 - 18 = 0$$

So, the solution is: $I_1 = \dfrac{5.5\text{V} - 18\text{V}}{25\Omega} =$ **- 0.50 A**

The minus sign in the answer indicates that the original assumption of the direction of the current was wrong, so the current flows from right to left through resistor R_1.

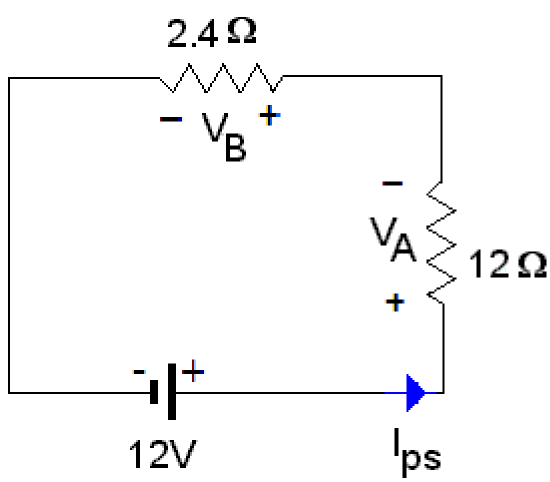
Problem 8.- In the following circuit, determine values of I_{ps}, I_1, I_2, V_A and V_B.



Solution: Notice that the 3-ohm resistor is in parallel with the 12-ohm resistor (they share the same voltage) so they are equivalent to one resistor with a value:

$$R_{equivalent} = \frac{1}{\frac{1}{3\Omega} + \frac{1}{12\Omega}} = 2.4\Omega$$

The circuit looks like this:



The current through the power supply can be calculated by applying Ohm's law to the two resistors that are now in series, so:

$$I_{ps} = \frac{12V}{2.4\Omega + 12\Omega} = \frac{12V}{14.4\Omega} = \mathbf{0.833\ A}$$

With this value of the current we can calculate the voltages:

$$V_A = 12\Omega(0.833A) = \mathbf{10\ V}$$

$$V_B = 2.4\Omega(0.833A) = \mathbf{2\ V}$$

To determine the currents I_1 and I_2 we can use Ohm's law one more time with the value of the voltage V_B.

$$I_1 = \frac{2V}{12\Omega} = \mathbf{0.167\ A}$$

$$I_2 = \frac{2V}{3\Omega} = \mathbf{0.666\ A}$$