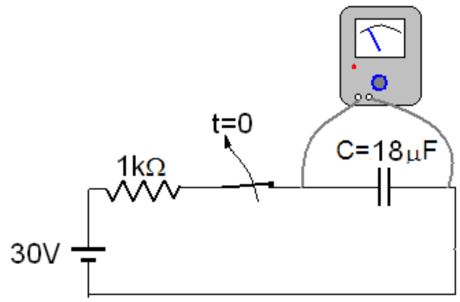
Physics II

RC Circuits

Problem 1.- The figure depicts the circuit used in the lab “RC circuits”. The switch in the figure has been closed for a long time, so the capacitor is charged to 30 volts. Then at time t=0 you open the switch, and the voltage starts dropping (the multimeter behaves as a resistance of 1.0MΩ) find how long it takes for the voltage in the capacitor to get to 4.06 volt.



Solution: According to what we learned in theory and confirmed in experiments, the equation that describes the discharge of the capacitor is:

$V_C = V_o e^{-t/\tau}$ where $\tau = RC$, so with the values of the problem:

$\tau = RC = (1.0M\Omega)(18\mu F) = 18s$

and $4.06 = 30e^{-t/18s} \rightarrow \frac{4.06}{30} = e^{-t/18s} \rightarrow \ln\left(\frac{4.06}{30}\right) = -t/18s \rightarrow t = -18\ln\left(\frac{4.06}{30}\right) = \mathbf{36s}$

Problem 2.- A 47µF capacitor is charged to an initial potential of V_o=30 volts, then it discharges through a 10 MΩ resistor. Calculate how long it takes to reach 11 volts.

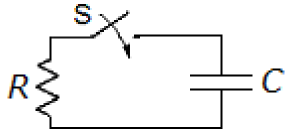
Solution: The time constant is $\tau = RC = (10M\Omega)(47\mu F) = 470\text{seconds}$

The equation for discharge is: $V_c = V_o e^{-t/\tau}$

With the values of the problem: $11 = 30e^{-t/470}$

Solving for t we get: $\frac{11}{30} = e^{-t/470} \rightarrow \ln\left(\frac{11}{30}\right) = -t/470 \rightarrow t = -470\ln\left(\frac{11}{30}\right) = \mathbf{472\ s}$

Problem 3.- The RC circuit of the following figure has R=33kΩ and C=47µF. The capacitor is initially charged to a voltage V_o=100volts. Determine how long it will take after closing the switch S for the voltage to drop to 1 volt.



Solution: We use the discharge equation: $V_C = V_o e^{-\frac{t}{RC}}$

$$1 = 100^{-\frac{t}{(33000)(47\times10^{-6})}} \rightarrow 0.01 = e^{-\frac{t}{(33000)(47\times10^{-6})}} \rightarrow \ln(0.01) = -\frac{t}{(33000)(47\times10^{-6})}$$

$$\rightarrow t = -(33000)(47\times10^{-6})\ln(0.01) = \mathbf{7.1\ s}$$

Problem 4.- Suppose you have an oscillator whose period is determined by an RC circuit (like the siren built in the lab). What should you do if you want to increase the period?

A) Increase the resistance.
B) Increase the capacitance.
C) Reduce the capacitance.
D) Reduce the resistance.
E) A or B
F) C or D

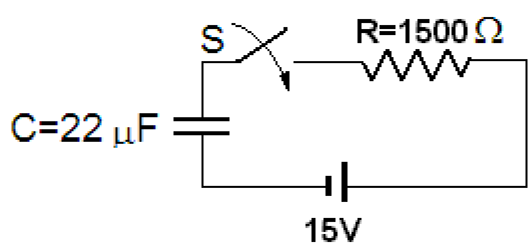
Give a short rationale for your choice.

Solution: To increase the period:

A) Increase the resistance.
OR
B) Increase the capacitance.

Because $\tau = RC$, increasing either R or C or both will increase the time constant. And if the period of the oscillator is proportional to the time constant, it will increase as well.

Problem 5.- Find how long it will take for the voltage in the resistor to drop to 10 V after closing the switch in the following circuit:



Solution: The current in an RC circuit, like the one in the problem, is given by:

$$I = I_o e^{-\frac{t}{RC}}$$

The product RC is a constant, sometimes it is represented by the Greek letter "τ". It is also called the "time constant" of the circuit. In our problem it is:

$$\tau = RC = (22\mu F)(1500\Omega) = 0.033s$$

So, the current is given by $I = I_o e^{-\frac{t}{0.033s}}$

The value of I_o is the maximum current, which happens when the switch just closed (at t=0). At that point there is no voltage in the capacitor and all the voltage of the source is across the resistor, so:

$$I_o = \frac{V_{source}}{R} = \frac{15V}{1500\Omega} = 0.010A$$

The question is, when is that the voltage in the resistor will be 10V? At that point the current will be 10V/1500Ω=0.0066 A So:

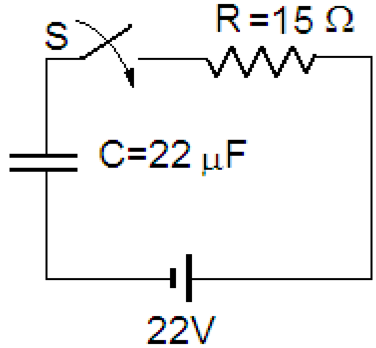
$$0.0066 = 0.010e^{-\frac{t}{0.033s}}$$

Dividing both sides of the equation by 0.010: $\rightarrow 0.66 = e^{-\frac{t}{0.033s}}$

Taking natural logarithm to both sides of the equation $\quad \text{Ln}(0.66) = -\frac{t}{0.033s}$

Solving for t $\quad \rightarrow t = -0.033s[\text{Ln}(0.66)] =$ **0.014 s**

Problem 6.- The capacitor was initially uncharged. Find how much time has to pass after closing the switch in the circuit shown for the current to drop to 1mA.



Solution: The current after closing the circuit, assuming the charge in the capacitor was zero at t=0, is given by:

$$I(t) = \frac{V}{R}e^{-\frac{t}{RC}}$$

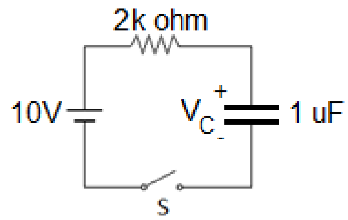
We want the time when this current is 1mA, so:

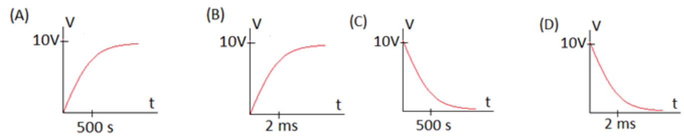
$$1mA = \frac{V}{R}e^{-\frac{t}{RC}} \rightarrow e^{-\frac{t}{RC}} = 1mA\frac{R}{V} \rightarrow -\frac{t}{RC} = \ln(1mA\frac{R}{V}) \rightarrow t = -RC\ln(1mA\frac{R}{V})$$

Plugging-in the values of the problem:

$$t = -RC\ln(1mA\frac{R}{V}) = -(15\Omega)(22\times10^{-6}F)\ln(1mA\frac{15\Omega}{22V}) = \mathbf{2.4ms}$$

Problem 7.- In the RC circuit shown below, the capacitor is initially discharged and switch S is closed at time t=0. Which of the graphs best describes the voltage in the capacitor V_C?





Solution: Since the capacitor is initially discharged, its voltage is zero and it will increase according to the equation:

$$V_C = 10\left(1 - e^{-t/\tau}\right)$$

Where $\tau = RC = 2k\Omega \times 1\mu F = 2\text{ms}$

Answer: **B**