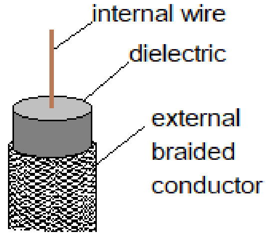
Physics II

Resistivity

Resistivity of copper $\rho_{Cu} = 1.68\times10^{-8}\,\Omega m$

Problem 1.- A coaxial cable has an internal copper wire of 0.5mm diameter and an outer cylindrical braided conductor of diameter 5.5mm. Calculate the resistance between the two conductors, if the length of the wire is 20m and the resistivity of the dielectric in the gap between the conductors is 250,000 Ωm.

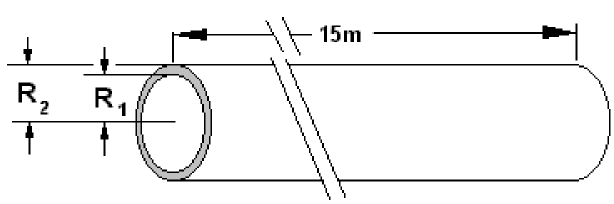


Solution: Consider the equation $dR = \rho\dfrac{dr}{A}$, where "dr" is the differential in the direction of charge flow. In this case, it is a radial flow. The area in the denominator of that equation is the area that the charge flows through and in this case is the lateral area of a cylinder whose length is 20m and whose radius is variable from 0.5mm to 5.5mm.

So: $dR = \rho\dfrac{dr}{2\pi r(20)}$ and integrating:

$$R = \int_{0.0005}^{0.0055} \rho\frac{dr}{2\pi r(20)} = \frac{\rho}{2\pi(20)}\ln\left(\frac{0.0055}{0.0005}\right) = \mathbf{7.77\ k\Omega}$$

Problem 1a.- What is the electric resistance ***radially*** through a pipe made of copper that has a length of 15m, internal radius R_1=10mm and external radius R_2=12mm.



Problem 2.- Find the resistance of a tapered copper wire that has a radius of 1mm on one end and 1.5mm on the other. Consider the length of the wire to be 10m.

Solution: The radius of the wire is

$$r = 0.001 + 0.00005x$$

Where x is the distance measured from one end.
The cross-section area is

$$A = \pi r^2 = \pi(0.001 + 0.00005x)^2$$

The differential of resistance is then: $dR = \frac{\rho dx}{A} = \frac{\rho dx}{\pi(0.001 + 0.00005x)^2}$

Integrating:

$$R = \int_0^{10} \frac{\rho dx}{\pi(0.001 + 0.00005x)^2} = -\left.\frac{\rho}{\pi(0.001 + 0.00005x)0.00005}\right|_0^{10}$$

$$R = \frac{\rho}{\pi 0.00005}\left(-\frac{1}{(0.0015)} + \frac{1}{(0.001)}\right) = \mathbf{0.0354\ \Omega}$$

Problem 2a.- A wire is uniformly tapered from a radius R_1=1mm to a final radius R_2=3.0 mm. The length of the wire is 20 m, so the equation that describes the radius is

$$r = 0.001 + 0.0001x$$

Calculate the resistance of the wire.

Solution: By integration $R = \int_0^L \rho \frac{dx}{\pi\left(a + \frac{b-a}{L}x\right)^2}$

$R = \rho \frac{L}{\pi ab}$, where a and b are the radii at the two ends.

$$R = 1.67 \times 10^{-8} \frac{20}{3.1416(0.001)(0.003)} = \mathbf{0.0354\ \Omega}$$

Problem 3.- Two wires, one of copper and one of aluminum have the same length and same electric resistance. Which one is thicker? why?

Solution: Aluminum must be thicker to compensate for its higher resistivity.

Problem 4.- Find the resistance of a 25m length of copper wire of diameter 2.85 mm and compute the voltage drop when carrying a current of 12A.

Solution: To find the resistance we use the equation

$$R = \rho \frac{L}{A},$$

Where ρ is the resistivity, L is the length and A is the area of cross section of the wire, which is $\frac{\pi d^2}{4}$, d being the diameter.

So, $$R = \rho \frac{L}{\left(\frac{\pi d^2}{4}\right)} = \frac{4L\rho}{\pi d^2}$$

With the values given in the problem:

$$R = \frac{4(1.67\times10^{-8}\Omega m)(25m)}{(3.1416)(0.00285m)^2} = \mathbf{0.0654\ \Omega}$$

To find the drop in voltage we use Ohm's law:

$$V = RI = (0.0654\ \Omega)(12\ A) = \mathbf{0.785\ V}$$

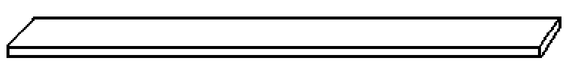
Problem 4a.- A coil is made with 10 meters of copper wire with diameter 0.65 mm. Calculate the voltage drop in the wire with a current of I=2.5 Amps.

Solution: The calculation is like the one we did in the previous problem:

$$R = \rho \frac{L}{A} = 1.67\times10^{-8} \frac{10}{\pi(0.65\times10^{-3})^2/4} = \mathbf{0.503\ \Omega}$$

The voltage drop is $V = IR = 0.51\times2.5 = \mathbf{1.26\ V}$

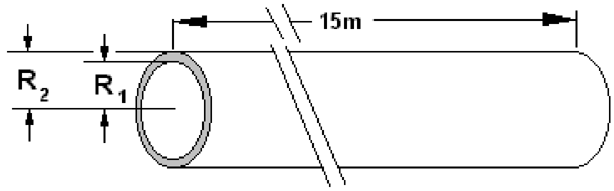
Problem 5.- What is the electric resistance along a 20m-long bar of copper that has a thickness of 0.5cm and a width of 4cm.



Solution:

$$R = \rho \frac{L}{A} = 1.67\times10^{-8} \frac{20}{0.005\times0.04} = \mathbf{0.00167\ \Omega}$$

Problem 6.- What is the electric resistance along a pipe made of copper that has a length of 15m, internal radius $R_1 = 10mm$ and external radius $R_2 = 12mm$.



Solution:

$$R = \rho \frac{L}{A} = 1.68 \times 10^{-8} \frac{15}{\pi(0.012^2 - 0.010^2)} = \mathbf{0.0018\ \Omega}$$

Problem 7.- Explain the reason why metals have higher resistance at higher temperatures.

Solution: In metals, when you increase the temperature, you increase the frequency of collisions between electrons and defects, other electrons and impurities, so the resistance increases.

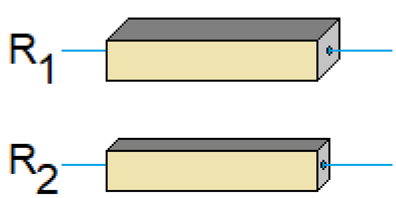
Problem 8.- What is superconductivity?

Solution: Superconductivity is a phenomenon that occurs in certain metals and compounds, where the resistivity becomes zero at low temperatures.

Problem 9.- In what units do you measure resistivity, electric field and electric potential?

Solution: Resistivity (ρ not R) is measured in Ωm. Electric field is measured in N/C or V/m. Electric potential is measured in volt (V).

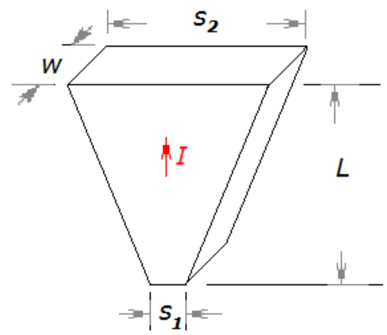
Problem 10.- Two resistors R_1 and R_2 are made of the same material and have the same length, but R_2 has half the cross section than R_1. If they are connected in parallel to a battery and the power consumed by R_1 is 1W, what is the power consumed by R_2?



Solution: R_2 is twice R_1 because it has the same length, but half the cross section. When connecting it in parallel with R_1 they will be at the same voltage and since the power is V^2/R, R_2 will consume 0.5W

Problem 11.- An atomic microscope has a tip made of copper foil (resistivity ρ_{Cu}) cut in the shape of a triangle as shown in the figure. Calculate the resistance of the tip assuming the current flows uniformly through its cross section from the small rectangular end to the large one.

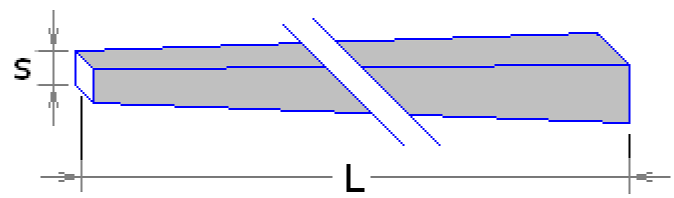
Hint: The width of the cross section can be written as: $s = s_1 + \left(\frac{s_2 - s_1}{L}\right)x$, where x is the distance to the end of the tip.



Solution:

$$R = \int_0^L \rho_{Cu} \frac{dL}{Area} = \int_0^L \rho_{Cu} \frac{dx}{\left(s_1 + \left(\frac{s_2 - s_1}{L}\right)x\right)W} = \frac{\rho_{Cu}}{W}\frac{L}{s_2 - s_1}\ln\left(\frac{s_2}{s_1}\right)$$

Problem 12.- Find the resistance of a copper connector whose shape has a square cross section. The conductor is tapered, so one end has a side s=1mm and the other s=1.5mm. Consider the length of the wire to be 10m and resistivity of copper $\rho_{Cu} = 1.68 \times 10^{-8}\,\Omega m$



Hint: Notice that the side "s" as a function of the distance to the smaller end is:
$s = 0.001 + 0.00005x$

Solution: $R = \int \frac{\rho dL}{A} = \int_0^{10} \frac{\rho dx}{\left(s_1 + \left(\frac{s_2 - s_1}{L}\right)x\right)^2}$

We change variable to $s = s_1 + \left(\frac{s_2 - s_1}{L}\right)x = 0.001 + 0.00005x$, so:

$$R = \int_0^{10} \frac{\rho dx}{\left(s_1 + \left(\frac{s_2 - s_1}{L}\right)x\right)^2} = \frac{\rho L}{s_2 - s_1}\int_{s1}^{s2} \frac{dx}{s^2} = \frac{\rho L}{s_2 - s_1}\left(-\frac{1}{s_2} + \frac{1}{s_1}\right) = \frac{\rho L}{s_2 s_1}$$

$$R = \frac{\rho L}{s_2 s_1} = \frac{1.68\times10^{-8}\,\Omega m\times10}{(0.001)(0.0015)} = \mathbf{0.112\ \Omega}$$

Problem 13.- Determine a formula for the total resistance of a spherical shell made of a material whose resistivity is ρ and whose inner and outer radius are r_1 and r_2. Assume the current flows radially outward.

Solution: If we consider a thin shell, we can write:

$$dR = \rho\frac{dr}{4\pi r^2}$$ Where r is he radius of the thin shell.

The value of *r* varies from r_1 to r_2, so the total resistance is:

$$R = \int_{r_1}^{r_2} \rho\frac{dr}{4\pi r^2} = -\frac{\rho}{4\pi r}\bigg|_{r_1}^{r_2} = \frac{\rho}{4\pi r_1} - \frac{\rho}{4\pi r_2} = \frac{\rho(r_2 - r_1)}{4\pi r_1 r_2}$$