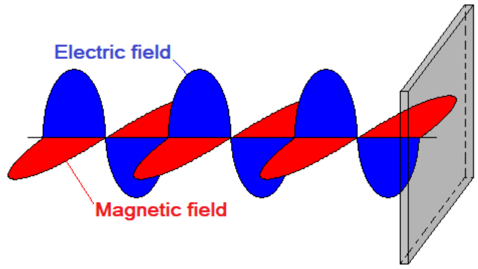
Physics II

EM Wave Intensity



The intensity of an electromagnetic wave is defined as power per area, or energy per time per area.

$$I = \frac{P}{A} = \frac{E}{tA}$$

The intensity of an electromagnetic wave is given by:

$I = \varepsilon_o c E_{rms}^2$ Where E**rms** is the **r**oot **m**ean **s**quare value of the ***electric field***

E stands for electric field, not energy, in this equation. If you use the peak value instead of the rms value, you need to divide the equation by 2.

$c = 2.998 \times 10^8$ m/s speed of light in vacuum

$\varepsilon_o = 8.85\times10^{-12}$ F/m Permittivity of free space

Problem 1.- A laser delivers 15 mJ to an area of 1 mm^2 in a pulse that lasts 10 ns. Calculate the amplitude of the electric field.

Solution: The intensity of the laser is $I = \frac{15\text{mJ}}{(1\text{mm}^2)(10\text{ns})} = 1.5\times10^{12}\,\frac{\text{J}}{\text{m}^2\text{s}}$

To find the electric field recall that $I = \frac{1}{2}\varepsilon_o c E^2$, so:

$$E = \sqrt{\frac{2I}{\varepsilon_o c}} = \sqrt{\frac{2(1.5\times10^{12}\text{J/m}^2\text{s})I}{(8.85\times10^{-12}\text{F/m})(3\times10^8\text{m/s})}} = \mathbf{3.36\times10^7\ V/m}$$

Problem 2.- A pulsed excimer laser emits 0.135J of energy in a pulse that lasts 15ns and it is focused over an area of 2.25mm^2. Calculate the amplitude of its magnetic field.

Solution: The power of the laser is: $P=\frac{\text{Energy}}{\text{time}}=\frac{0.135\text{J}}{15\times10^{-9}\text{s}}=9\text{MW}$

The intensity is: $I=\frac{\text{Power}}{\text{Area}}=\frac{9\text{MW}}{2.25\times10^{-6}\text{m}^2}=4\times10^{12}\text{W/m}^2$

The electric field is:

$$I=\frac{1}{2}\varepsilon_o cE^2 \rightarrow E=\sqrt{\frac{2I}{\varepsilon_o c}}=\sqrt{\frac{2(4\times10^{12}\text{W/m}^2)}{(8.85\times10^{-12}\text{F/m})(3\times10^8\text{m/s})}}=\mathbf{55MV/m}$$

And the magnetic field:

$$B=\frac{E}{c}=\frac{55\text{MV/m}}{3\times10^8\text{m/s}}=\mathbf{0.183\ T}$$

Problem 3.- If the amplitude of the electric field of an EM wave is 5.5 V/m, (a) Calculate the amplitude of the magnetic field and (b) Find the average intensity (power per unit area) of the wave.

Solution:

a) The amplitude of the magnetic field is given by: $B=\frac{E}{c}=\frac{5.5V/m}{3\times10^8 m/s}=\mathbf{1.83\times10^{-8}\ T}$

b) The average intensity is given by:

$$\bar{I}=\frac{c\varepsilon_o E^2}{2}=\frac{(3\times10^8 m/s)(8.85\times10^{-12}F/m)(5.5V/m)^2}{2}=\mathbf{0.040\ W/m^2}$$

Problem 4.- If the average intensity of an EM wave is 1.5×10^{-3}W/m^2, calculate the amplitudes of the electric and magnetic fields. [Z_o=377Ω].

Solution: The average intensity is given by:

$$\bar{I}=\frac{E^2}{2Z_o}\rightarrow E=\sqrt{2Z_o\bar{I}}=\sqrt{2(377\Omega)(1.5\times10^{-3}W/m^2)}=1.06V/m$$

The amplitude of the magnetic field is given by: $B=\frac{E}{c}=\frac{1.06V/m}{3\times10^8 m/s}=3.5\times10^{-9}$ T

Problem 5.- Based on your newly acquired knowledge of electromagnetic waves, why would you say that water is heated in a microwave oven, but air not so much?

Solution: Water molecules are polar (they have a dipole moment), so they couple easily to the electric field of the electromagnetic waves. That is not the case with O_2, N_2 and Ar which do not have dipole moments.

Problem 6.- An FM station broadcasts with a power of 1kW in all directions. How much is the amplitude of the electric field when detected by a car radio antenna 2.3 km away from the station?