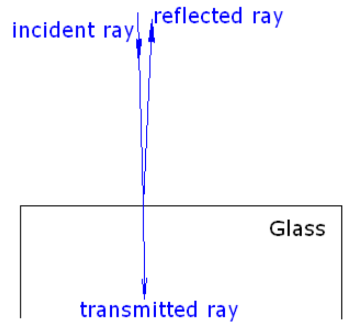
Physics II

Fresnel Equations

Problem 1.- One important application of Fresnel equations is in the calculation of reflected intensity. Consider the following: A light ray of intensity **I=1W/m²** is incident normal ($\theta_i=0$) on glass that has an index of refraction **n= 1.48… 1.7**
Calculate how much intensity is reflected.



Note: For your calculation notice that the intensity of the reflected ray is proportional to the electric field squared, so:

$$\frac{I_{reflected}}{I_{Incident}} = \left(\frac{E_{reflected}}{E_{Incident}}\right)^2$$

Also notice that since the incident angle is $\theta_i=0$ it doesn't matter if you use the TE or TM mode equations, they will give you the same value.

Solution:

The ratio of the electric field reflected, and incident is: $\frac{E_r}{E_i} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$

Since the angle of incidence is zero, the transmitted angle is also zero and the equation can be written as: $\frac{E_r}{E_i} = \frac{n_i - n_t}{n_i + n_t}$, moreover the index of refraction of air is almost 1, so the ratio is $\frac{E_r}{E_i} = \frac{1-n_{glass}}{1+n_{glass}}$

For our calculation the intensity of the reflected ray is proportional to the electric field squared, so:

$$\frac{I_{reflected}}{I_{Incident}} = \left(\frac{1-n_{glass}}{1+n_{glass}}\right)^2$$

The larger the mismatch between indexes of refraction the higher the percentage of light reflected, here is a table:

n-glass	Intensity Reflected	
1.48	37.5	mW/m^2
1.5	40.0	mW/m^2
1.52	42.6	mW/m^2
1.54	45.2	mW/m^2
1.56	47.9	mW/m^2
1.58	50.5	mW/m^2
1.6	53.3	mW/m^2
1.62	56.0	mW/m^2
1.64	58.8	mW/m^2
1.66	61.6	mW/m^2
1.68	64.4	mW/m^2
1.7	67.2	mW/m^2

Problem 2.- What fraction of the incident light intensity (irradiance) will be reflected in the TE mode when the angle of incidence is 45º on an air-water interface?
$n_{water} = 1.33$

Solution: Let's find the angle of transmission first:

$$n_{air} \sin 45^\circ = n_{water} \sin \theta_t \rightarrow \theta_t = 32.11^\circ$$

Now we use Fresnel's equations for the TE mode:

$$\frac{E_r}{E_i} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{1\cos 45^\circ - 1.33\cos 32.11^\circ}{1\cos 45^\circ + 1.33\cos 32.11^\circ} = -0.229$$

And the ratio of intensities is equal to the ratio of electric fields squared:

$$\frac{I_r}{I_i} = \left(\frac{E_r}{E_i}\right)^2 = 0.229^2 = \mathbf{0.052}$$