Physics II

Poynting Vector

Poynting vector $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_o}$

Problem 1.- The value of the electric field of an electromagnetic wave is $\vec{E} = 9\hat{x} + 12\hat{y}$ in volts per meter, while the magnetic field is $\vec{B} = 5 \times 10^{-8}\,\hat{z}$ in tesla. Find the direction of the velocity of this wave and its instantaneous intensity.

Solution: $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_o} = \dfrac{1}{4\pi \times 10^{-7}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 9 & 12 & 0 \\ 0 & 0 & 5 \times 10^{-8} \end{vmatrix} = \dfrac{10^{-8}}{4\pi \times 10^{-7}} (60, -45, 0)$

The magnitude of the vector is:

$$\left|\vec{S}\right| = \frac{\sqrt{60^2 + 45^2}}{4\pi \times 10} = \mathbf{0.6\ W/m^2}$$

Problem 2.- If a radio transmitter has a vertical antenna, should a receiver antenna (rod type) be vertical or horizontal to get the best reception? Why?

Solution: The best coupling to the electromagnetic wave will be when the antenna is parallel to the electric field, so it should be a **vertical** rod.

Problem 3.- When calculating the Poynting vector, what is the meaning of its magnitude and direction?

Solution: The magnitude of the Poynting vector is equal to the intensity of the electromagnetic wave and its direction is the direction of propagation.

Problem 4.- A pulsed laser emits 0.135J of energy in a pulse that lasts 15ns and it is focused over an area of 2.25mm^2. Calculate the amplitude of its magnetic and electric fields.

Solution: The power of the laser is: $P = \dfrac{E}{t} = \dfrac{0.135J}{15 \times 10^{-9} s} = 9MW$ (E is energy in this case)

The intensity is: $I = \dfrac{P}{A} = \dfrac{9MW}{2.25 \times 10^{-6} m^2} = 4 \times 10^{12} W/m^2$

The electric field is:

$$I = \frac{1}{2}\varepsilon_o c E^2 \rightarrow E = \sqrt{\frac{2I}{\varepsilon_o c}} = \sqrt{\frac{2(4\times10^{12} W/m^2)}{(8.85\times10^{-12} F/m)(3\times10^8 m/s)}} = \mathbf{55MV/m}$$

And the magnetic field:

$$B = \frac{E}{c} = \frac{55MV/m}{3\times10^8 m/s} = \mathbf{0.183\ T}$$