Physics II

Circular Apertures

(And the limits of resolution)

Limit of resolution:

$\theta = \sin^{-1}\left(1.22\frac{\lambda}{D}\right)$, where D is the diameter of the aperture.

Problem 1.- The Hubble space telescope has a diameter of D=2.4 meters.

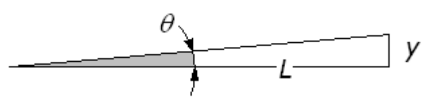
(a) Calculate the maximum angular resolution.
Take the wavelength of visible light as $\lambda = 550nm$

(b) Knowing the angular resolution estimate the minimum separation between two light sources that can be distinguished with the telescope 1 mile away [1 mile is 1609 m]

Solution: The resolution for a circular aperture:

$\theta = \sin^{-1}\left(1.22\frac{\lambda}{D}\right) = \sin^{-1}\left(1.22\frac{550nm}{2.4}\right) =$ **1.6×10^{-6} degrees**

For the distance between the light sources:
Basic geometry equation: $y = L\tan\theta = 1609\tan(1.6\times10^{-6}) =$ **0.44 mm**



Problem 2.- A paparazzo camera has an objective lens 12cm in diameter.

a) Calculate the minimum angle between two points that will still be distinguished in the picture. Consider that the wavelength is $\lambda = 533nm$

b) Knowing the minimum angle and knowing that the distance from the two points to the camera is 150m, how close can the points be and still be distinguished?

Solution:

a) $\sin\theta = \frac{1.22\lambda}{Diameter} = \frac{1.22\times533\times10^{-9}}{0.12} = 5.42\times10^{-6} \rightarrow \theta = 3.1\times10^{-4}$ degrees

b) $x = L\sin\theta = 150\times5.42\times10^{-6} = 0.8$ mm

Problem 3.- A 10-inch telescope is advertised as having "diffraction limited optics" which means that the only limitation to the image sharpness comes from diffraction. Assuming the ad is accurate; determine the minimum angle of separation between double stars that can be resolved with the instrument. Take the wavelength of light as 600 nm.
[1 inch = 0.0254 m]

Solution: The criterion is that if the two stars are separated by more than the radius of the “Airy disk” then we can distinguish the two sources:

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{D}\right) = \sin^{-1}\left(\frac{1.22 \times 600 \times 10^{-6}}{10 \times 0.0254}\right) = \mathbf{0.00017^o}$$

Problem 4.- Why do we see a bright spot in the middle of circular shadows?

Solution: Such a bright spot is the called Poisson’s dot. It might be hard to see, but its explanation is fascinating. It played a role in accepting the theory that light has wave properties.
Light from any point on the edge of an opaque circle covers the same distance to the center of the round shadow, so the waves add constructively, producing the bright spot.

Problem 5.- Calculate the minimum diameter of a telescope that you need to resolve the separation between Pluto and its moon Charon if the angular separation is 0.000153° as seen from Earth during the observation.
Assume perfect conditions for seeing, diffraction limited optics and wavelength of the light 600nm.

Solution: The diffraction limit is given by: $\sin\theta = \dfrac{1.22\lambda}{D}$, so the diameter needed is:

$$D = \frac{1.22\lambda}{\sin\theta}$$

For an angle of 0.000153° we get: $D = \dfrac{1.22 \times 600nm}{\sin 0.000153^\circ} = \mathbf{0.27\ m}$

A larger telescope is necessary because seeing conditions are not perfect, light pollution washes out the images and the equation does not work well when the sources have different intensities (Pluto is brighter than Charon).