# Physics II

## Diffraction Grating

Diffraction grating equation: $\theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$

**Problem 1.-** Determine the angular positions of the two strongest hydrogen lines:

$\lambda_{RED} = 656nm$
$\lambda_{BLUE-GREEN} = 486nm$

if they are observed with a diffraction grating that has 14,000 lines per inch.

**Solution**: According to the problem: $d = \frac{0.0254}{14,000} = 1.81\mu\text{m}$

We use the equation $\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right)$ to find the angles.

For red we get: $\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{656nm}{1.81\mu m}\right) =$ **21.2°**

For blue-green: $\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{486nm}{1.81\mu m}\right) =$ **15.6°**

**Problem 2.-** A diffraction grating has lines separated by 3.5µm. Calculate the angle of diffraction of the first order fringe for green light (λ=530nm) and red light (λ=650nm)

**Solution:** The equation that describes diffraction gratings is:

$dSin\theta = m\lambda$

If we are looking for the first fringe m=1, so:

$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$

For the given wavelengths we get:

(a) Green light $\theta = \sin^{-1}\left(\frac{530nm}{3.5\mu m}\right) =$ **8.7°**

(b) Red light $\theta = \sin^{-1}\left(\frac{650nm}{3.5\mu m}\right) =$ **10.7°**

**Problem 3.-** A diffraction grating is used to diffract the light emitted by a flame spectrometer and detect sodium (wavelength of 589nm). At what angle should we set the detector if the line density of the grating is 20,000 lines per inch? [1 inch = 2.54cm].

**Solution:**

$$d = \frac{0.0254m}{20{,}000} = 1.27\times10^{-6}m$$

$$\theta = \sin^{-1}\left(\frac{589\times10^{-9}}{1.27\times10^{-6}}\right) = \mathbf{27.6^{o}}$$

**Problem 3a.-** The sodium line is a doublet. Calculate the angular separation between the two lines (589.0 nm and 589.6 nm) if the grating has a density of 15,000 lines per inch. [1 inch = 2.54cm]

**Solution:** The density of lines implies a distance between slits of:

$$d = \frac{1inch}{15000lines} = \frac{2.54\times10^{-2}m}{15000lines} = 1.693\times10^{-6}\text{ m}$$

To get the angular separation we will find the angle for each wavelength and calculate the difference:

$$d\sin(\theta) = \lambda \rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

$$\theta_1 = \sin^{-1}\left(\frac{589.0\times10^{-9}m}{1.693\times10^{-6}}\right) = 20.355^{o}$$

$$\theta_2 = \sin^{-1}\left(\frac{589.6\times10^{-9}m}{1.693\times10^{-6}}\right) = 20.376^{o}$$

So, the difference is **0.022º**. To give some idea of how small this value is, the diameter of the moon from our perspective is ~0.5º, so this separation is ~23 times smaller.

**Problem 4.-** One of the first diffraction gratings was made with a thin wire wrapped around two threaded rods. In his crude instrument there were 2000 threads per inch. Calculate the full angular span of visible light (400nm to 700nm) observed with this grating.