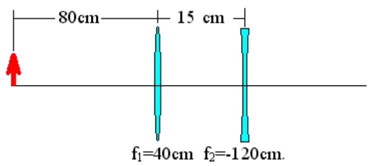
Physics II

Lenses

Lens and mirror equations: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Problem 1.- A convergent lens of focal length f_1=40cm is placed 15 cm in front of a divergent lens of focal length f_2=-120cm. Calculate the final position of the image generated by this combination if the object is 80cm to the left of the convergent lens.



Solution: We solve the problem in two steps. Consider the converging lens first, where $d_{o1} = 80cm$ and $f_1 = 40cm$, so solving for d_{i1} gives:

$$\frac{1}{f_1} = \frac{1}{d_{o1}} + \frac{1}{d_{i1}} \rightarrow \frac{1}{40} = \frac{1}{80} + \frac{1}{d_{i1}} \rightarrow d_{i1} = 80$$

Now let's consider the second lens. The image generated by the first lens is 80cm to the right of the converging lens, so 65cm to the right of the divergent lens. This is considered a negative object distance, so $d_{o2} = -65cm$ and solving for d_{i2}

$$\frac{1}{f_2} = \frac{1}{d_{o2}} + \frac{1}{d_{i2}} \rightarrow \frac{1}{-120} = \frac{1}{-65} + \frac{1}{d_{i2}} \rightarrow d_{i2} = \textbf{142 cm}$$

Problem 2.- Alice is nearsighted with a far point of 25cm (she cannot see clearly beyond this point). What kind of glasses and of what power does she need to see distant objects clearly? [Neglect eye-lens distance in this problem].

Solution: To bring distant objects (do=∞) to the far point we need di to be equal to -0.25m, the minus sign because we want the image in front of the person, not behind her head. So the power will be:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{-0.25m} = \textbf{-4D}$$

Problem 3.- A nearsighted person has the far point at 0.66m (only objects closer than 66cm in front of the eye are seen clearly). Calculate the focal length of a corrective lens that will put the image of a very distant object ($d_o=\infty$) at the far point.
Ignore the distance eye-lens.

Solution: The object you want to see is very far away, so $d_o=\infty$, and the image should be at 0.66m *in front* of your eyes, which is considered negative for lenses, so $d_i=-0.66$m and the equation for the lens is:

$$\frac{1}{f}=\frac{1}{\infty}+\frac{1}{-0.6}\rightarrow f=\mathbf{-0.6m}$$

Problem 4.- We buy a magnifying glass that has a power of 2.5 diopters. If we place an object 30.0 cm away from the lens, where is the image formed (find d_i) and what kind of image is it (real, virtual, upright, inverted)?

Solution: A power of 2.5D means a focal length $f=\frac{1}{2.5D}=0.4m$. The problem says that the distance to the object is $d_o=0.3m$, so we can calculate the distance to the image:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\rightarrow\frac{1}{0.4m}=\frac{1}{0.3m}+\frac{1}{d_i}\rightarrow\frac{1}{d_i}=2.5-3.33=-0.833\rightarrow d_i=\mathbf{-1.2m}$$

Since d_i is negative the image is **virtual** (it is to the left of the lens), so we cannot project it on a screen or photographic film.

The magnification is $m=-\frac{d_i}{d_o}=-\frac{-1.2m}{0.3m}=+4$

Since the magnification is positive the image is **upright**.

Problem 4a.- We buy glasses that have a power of -1.5 diopters. If we use these lenses to observe an object that is 30.0 m away, where is the image formed and what kind of image is it (real, virtual, upright, inverted)?

Solution: The power of the glasses tells us the focal length:

$$\mathrm{f}=\frac{1}{-1.5}=-0.667\mathrm{m}$$

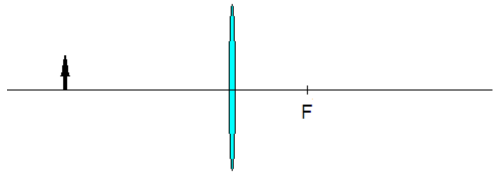
If the object is 30.0 m away it means that $\mathrm{d_o}=30.0\mathrm{m}$

We can find the image distance d_i with the equation:

$$\frac{1}{\mathrm{d_o}}+\frac{1}{\mathrm{d_i}}=\frac{1}{\mathrm{f}}\rightarrow\frac{1}{30.0\mathrm{m}}+\frac{1}{\mathrm{d_i}}=\frac{1}{-0.667\mathrm{m}}\rightarrow\mathrm{d_i}=-0.652\mathrm{m}$$

So, the image will be formed 0.625 m to the left of the lens (same side as the object). This is what we call a **virtual** image and since $\mathrm{m}=-\frac{\mathrm{d_i}}{\mathrm{d_o}}>0$ we know that the image is **upright.**

Problem 5.- In the following example of a converging lens find the image graphically and describe it (larger or smaller, virtual, or real, upright or inverted).



Solution: To find the image we draw two rays: One that is parallel to the horizontal which is refracted towards the focus and another that passes straight through the center of the lens:

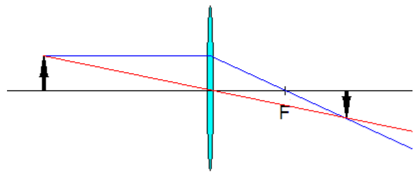
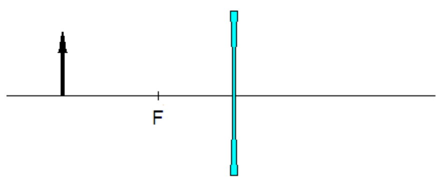
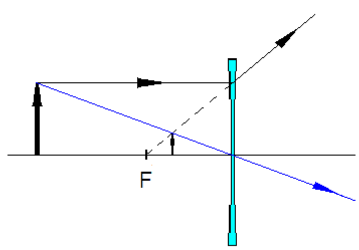


Image is smaller, real, and inverted.

Problem 5a.- In the following example of a diverging lens find the image graphically and measure the magnification "m".



Solution: To find the image we draw two rays: One that is parallel to the horizontal, which is refracted as if it were coming from the focus and another that passes straight through the center of the lens:



The intersection of the rays indicates the position of the image. To get the magnification we can measure the size of the image and divide by the size of the object. The magnification is positive in this case because the image is upright.

$$m \approx 0.33$$

Problem 6.- What is the position (d_i) and size (h_i) of the image of a 10-cm-high pencil located 1.2m from a lens with focal length f = +5cm?

Solution: First we find d_i:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\rightarrow\frac{1}{0.05}=\frac{1}{1.2}+\frac{1}{d_i}\rightarrow d_i=\mathbf{0.0522\ m}$$

Now we find h_i:

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o}\rightarrow\frac{h_i}{10cm}=-\frac{0.0522m}{1.2m}\rightarrow h_i=\mathbf{-0.43\ cm}$$

Problem 7.- What kind of lenses would you prescribe to a farsighted person, convergent or divergent? Why?

Solution: Farsighted people use **convergent lenses** to correct their problem. These lenses take close objects and create images farther away, beyond the "near point" of the eye.

Problem 7a.- A farsighted person has the near point at 45cm, what power of reading glasses would you prescribe so the person could read a book at 25 cm from his eye. Neglect the distance eye-lens.

Solution: You want to prescribe glasses that create a virtual image at 45 cm (so di=-0.45m) for an object located at 25cm (do=0.25m), so the power will be:

$$Power=\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{0.25}+\frac{1}{-0.45}=+1.78$$