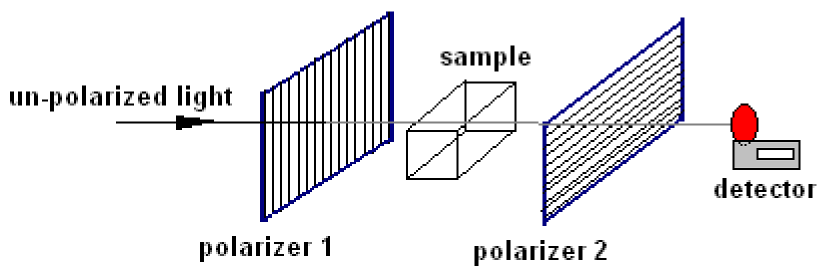
Physics II

Malus's Law

Polarizer equations:

i) The intensity of unpolarized light drops one half when passing through a polarizer
ii) The intensity of polarized light drops a factor of $\cos^2\theta$ when passing through a polarizer at an angle θ

Problem 1.- Two polarized films are rotated with respect to each other by 90º, so no light goes through them.
Then a sample of a crystal is put between the two films. The sample rotates the axis of polarization by 3º, without any loss of intensity. Find the fraction of the original intensity that is detected with the sample in place.



Solution: Let's analyze how the intensity changes after going through the polarizers and sample:

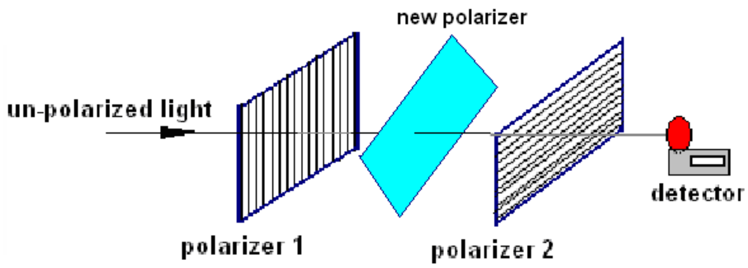
a) First polarizer: The intensity is cut in half because the polarizer only allows light of a definite polarization to go though. So, a factor of ½.

b) Sample: there is no loss in intensity according to the problem, which is reasonable if the crystal is transparent and has an antireflective coating. However, the angle of polarization is rotated 3 degrees.

c) Second polarizer. The intensity will have to be multiplied by $\cos^2\theta$, where θ is 90º-3º=87º (it was 90º without the sample, but it rotates the angle 3º).

So, the final intensity is: $\frac{1}{2}\cos^2(90^\circ - 3^\circ) = \mathbf{0.0014}$ of the original intensity

Problem 1a.- Two polarized films are rotated with respect to each other by 90 degrees, so no light goes through them.
Then a third polarizer is put in between the other two, so now a detector finds that 0.15% of the initial unpolarized light intensity goes through the three polarizers.
Find the angle of rotation between the first polarizer and the new one that was inserted.
[Note: there will be two solutions]



Problem 2.- Two polarizers reduce the intensity of incident unpolarized light to only 10%. Calculate the angle between the two polarizers.

Solution: The first polarizer reduces the intensity by a factor of ½ and the second polarizer by a factor of $\cos^2\theta$, so the final intensity is $\frac{1}{2}\cos^2\theta$ of the original value, but according to the problem this is 10%, so:

$$\frac{1}{2}\cos^2\theta = 0.1 \rightarrow \cos^2\theta = 0.2 \rightarrow \cos\theta = \sqrt{0.2} \rightarrow \theta = \cos^{-1}\left(\sqrt{0.2}\right) = \mathbf{63.4^o}$$

Problem 3.- Find how much intensity of a beam of un-polarized light will go through two polarizers that are rotated 60º with respect to each other.
Give your answer in percentage.

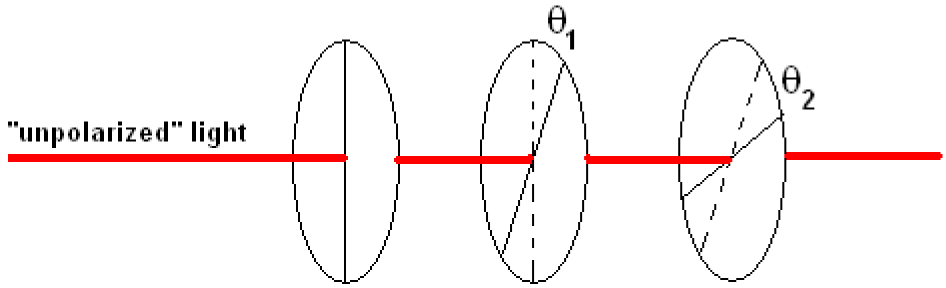
Solution: If you start with unpolarized light of intensity I_o, after the first polarizer you will have polarized light of intensity $\frac{1}{2}I_o$, the second polarizer is rotated, so there will be additional losses and the final intensity will be:

$$I_{FINAL} = \frac{1}{2}I_o[\text{Cos}^2(60^o)] = \frac{I_o}{8} = 12.5\% \text{ of } I_o$$

Problem 3a.- Find how much intensity of a beam of un-polarized light will go through two polarizers that are rotated 45º with respect to each other.

Solution: Half of the intensity of un-polarized light will go through the first polarizer, of this intensity a fraction equal to $\cos^2(45^o)=1/2$ will go through the second polarizer, so the final intensity will be only ¼ of the original intensity.

Problem 4.- Find how much intensity of a beam of un-polarized light will go through three polarizers, where the first and second are rotated $\theta_1=37^o$ with respect to each other and the second and third are rotated $\theta_2=30^o$ with respect to each other.



Solution: The first polarizer will reduce the intensity by a factor of 0.5 the second by a factor of $\cos^2 \theta_1$ and the third by $\cos^2 \theta_2$ where the cosine squared is Malus's law.

The final intensity for angles θ_1=37º and θ_2=30º is:

$0.5\cos^2 37º \cos^2 30º$ = **0.239**

The final intensity for angles θ_1=18º and θ_2=36º is:

$0.5\cos^2 18º \cos^2 36º$ = **0.296**