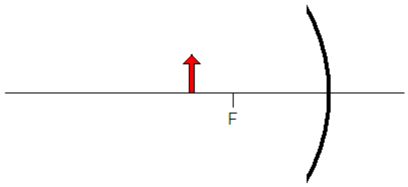
Physics II

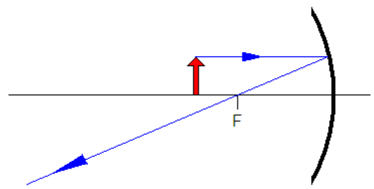
Mirrors

Lens and mirror equations: $$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \qquad m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

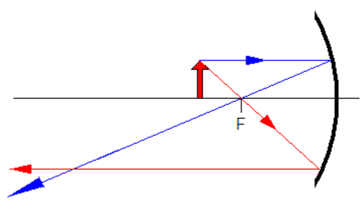
Problem 1.- In the following example of a concave mirror find the position of the image and describe it (virtual, real, upright, inverted, etc.)



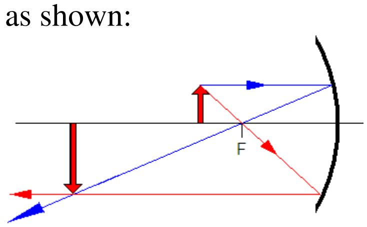
Solution: To find the image let's draw two light rays coming from the tip of the object:
The first light ray (shown in blue) that we can draw starts horizontally and is reflected towards the focal point:



A second light ray would be one that starts at the tip of the object and passes through the focal point (shown in red in the next figure). This ray will be reflected parallel horizontally:

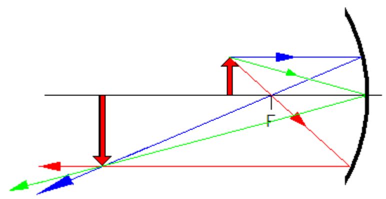


To an observer the light will seem to come from the intersection of the two light rays so the image will be formed at that point as shown:



The image is **larger, real, and inverted**.

Note: *Instead of one of the light rays chosen to solve this problem you could have used a light ray that starts at the tip of the object and goes to the vertex (ray shown in green). This ray will be reflected as if the mirror were flat and vertical.*



Problem 2.- You are standing 10.0 m from a convex security mirror in a store. Your image looks 0.25 times your actual size. What is the radius of curvature (R) of the mirror?

Solution: The object distance is +10.0 m and the magnification is 0.25, so

$$d_o = 10.0\text{m}$$

$$m = 0.25 = -\frac{d_i}{d_o} \rightarrow d_i = -2.5\text{m}$$

With the values of d_o and d_i we can calculate the focal length

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{10\text{m}} + \frac{1}{-2.5\text{m}} \rightarrow f = 3.33\text{m}$$

The radius of curvature (R) is twice the focal length, so **R = 6.66 m**

Problem 2a.- You are standing 4.0 m from a convex security mirror in a store. Your image looks 0.35 times your actual size. What is the focal length of the mirror?

Solution: Like the previous problem, the distance from the object (you) to the mirror is 4.0 meters and the magnification is 0.35

$d_o = 4.0$ and $m = 0.35$,

But $m = -\frac{d_i}{d_o}$, so we can find the distance to the image $0.35 = -\frac{d_i}{4.0} \rightarrow d_i = -1.4\text{m}$

And with these values we find the focal length

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \rightarrow \frac{1}{f} = \frac{1}{4.0} + \frac{1}{-1.4} = 0.25 - 0.714 = -0.464 \rightarrow f = \mathbf{-2.15\ m}$$

Problem 3.- A mirror at an amusement park shows an upright image of a person who stands 2.0 meters in front of it. If the image is 2.5 times the person's height, what is the radius of curvature of the mirror?

Solution: The problem says that $d_o = 2m$ and $m = 2.5$, so we can find the other values:

Using $m = -\frac{d_i}{d_o}$ we find d_i as follows:

$$2.5 = -\frac{d_i}{2m} \rightarrow d_i = -5m$$

Using $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ we find $f \rightarrow \frac{1}{f} = \frac{1}{2m} + \frac{1}{-5m} \rightarrow f = 3.33m$

The radius of the mirror is twice the focal length: $R = 2f =$ **6.66 m**

Problem 4.- What would we see on the moon if it had a polished mirror-like surface? Describe the image.

Solution: It would behave like a spherical convex mirror, so the images would be **upright, smaller, and virtual**.

Problem 5.- A solar cooker is made with a concave mirror. If the Sun's rays are focused 30cm in front of the mirror, what is the radius of the spherical surface from which the mirror was made?

Solution: If the parallel rays from the sun are focused 30cm in front of the mirror, its focal length is 30cm. For a spherical surface the radius is twice the focal length, so **R = 60 cm**.

Problem 6.- An object is placed 25 cm from a certain mirror and its image is half the size of the object, inverted and real. What is the focal length of the mirror and what kind of mirror is it?

Solution: Since the image is only half the size of the object and inverted, we know that $m = -\frac{1}{2}$

The object is 25 cm from the mirror, so $d_o = 25cm$. We can use these two equations to find d_i:

$$m = -\frac{d_i}{d_o} = -\frac{1}{2} \rightarrow d_i = \frac{d_o}{2} = \frac{25cm}{2} = 12.5cm$$

Next, we can find the focal length:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow f = \frac{1}{\frac{1}{25cm} + \frac{1}{12.5cm}} = 8.33cm$$

And since the focal length is positive this is a concave mirror.

Problem 7.- A shaving mirror is designed to magnify your face by a factor of 1.55 when your face is placed 25.0 cm in front of it.

(a) What kind of mirror is it?
(b) Describe the kind of image that it makes (virtual, real, upright, inverted, etc.) and
(c) Calculate the radius of curvature of the mirror.

Solution: The magnification is 1.55, so:

$$m = 1.55 = -\frac{d_i}{d_o}$$

And we know that $d_o = 25\text{cm}$, so:

$$m = 1.55 = -\frac{d_i}{d_o} \rightarrow d_i = -1.55(25\text{cm}) = -38.75\text{cm}$$

Using the equation for mirrors we can find the focal length:

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \rightarrow \frac{1}{f} = \frac{1}{25\text{cm}} - \frac{1}{38.75\text{cm}} \rightarrow f = 70.45\text{cm}$$

(a) The mirror is concave because the focal length is positive.

(b) The image is upright (since the magnification is positive) and it is a virtual image because it is behind the mirror (d_i is negative)

(c) The radius of curvature is twice the focal length, so it is 140.9 cm

Problem 8.- A dentist wants a small mirror that, when 2.55 cm from a tooth, will produce an upright image 5 times larger. What kind of mirror must be used and what would be its focal length?

Solution: The magnification is positive and equal to 5.0, so:

$$m = -\frac{d_i}{d_o} = 5.0$$

We also know the distance to the object: $d_o = 2.55\text{cm}$

Combining these two equations: $d_i = -5.0(2.55\text{cm}) = -12.75\text{cm}$

Moreover, using the mirror equation:

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \rightarrow \frac{1}{f} = \frac{1}{2.55\text{cm}} - \frac{1}{12.75\text{cm}} \rightarrow f = 3.19\text{cm}$$

Since the focal length is positive, this is a concave mirror.

Problem 9.- What is the focal length of a plane mirror? What is its magnification “m”?

Solution: A plane mirror can be thought of as a spherical mirror with an infinite radius or infinite focal length. With that in mind, the equations for mirrors and lenses don’t need to be changed for plane mirrors. You could also argue that there is no focal length at all because a plane mirror does not focus light.
The magnification of a flat mirror is m = +1, because the size of the image is the same as the size of the object and the image is upright.

Problem 10.- What is the focal length of a shiny Christmas ball that has a diameter of 10cm?

Solution: A shinny Christmas ball behaves like a convex mirror with focal length equal to half the radius. Since the diameter is 10 cm the radius is 5 cm and the focal length will be ***f* = -2.5cm**, where the minus sign is introduced by convention (convex mirrors have negative focal lengths).