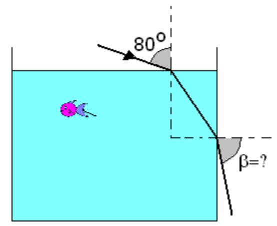
Physics II

Snell's Law

Snell's Law $n_i \sin\theta_i = n_r \sin\theta_r$

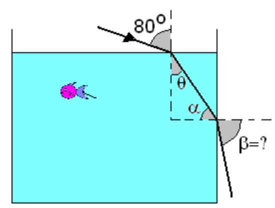
Problem 1.- You point a laser beam to the surface of a fish tank as shown in the figure. Find the angle β at which the beam exits through the side of the tank (you can ignore the effect of the glass wall). The index of refraction of water is 1.33



Solution: To solve the problem, we use Snell's law twice: at the first air-water interface:

$$n_{air}\sin 80^o = n_{water}\sin\theta$$

$$\rightarrow 1\sin 80^o = 1.33\sin\theta \rightarrow \sin\theta = \frac{\sin 80^o}{1.33} = 0.7405 \rightarrow \theta = \sin^{-1}(0.7405) = \mathbf{47.8^o}$$



Now, according to the geometry of the problem the incident angle on the second water-air interface is

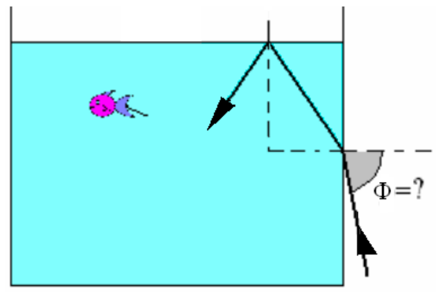
$$\alpha = 90^o - \theta = 42.2^o$$

And using Snell's law for a second time we get

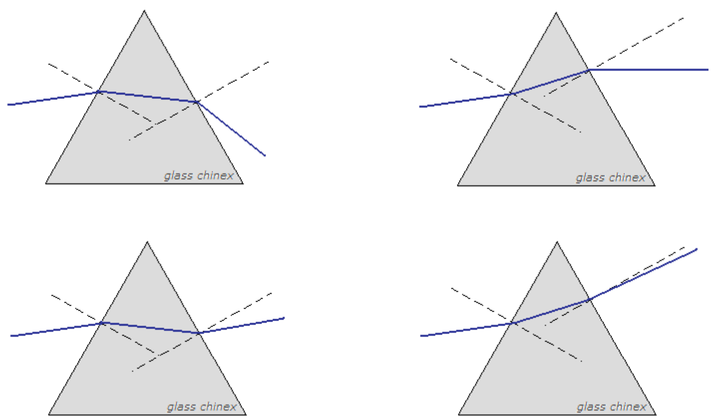
$$n_{water}\sin\alpha = n_{aire}\sin\beta$$

$$\rightarrow 1.33\sin 42.2^o = 1\sin\beta \rightarrow \sin\beta = 0.8939 \rightarrow \beta = \sin^{-1}(0.8939) = \mathbf{63^o}$$

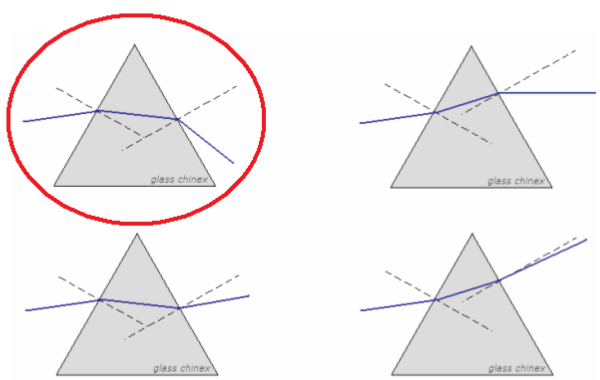
Problem 1a.- You point a laser beam to the surface of a fish tank as shown in the figure. Find the maximum angle Φ so the beam is totally internally reflected at the water-surface interface. You can ignore the effect of the glass wall.
The index of refraction of water is 1.33



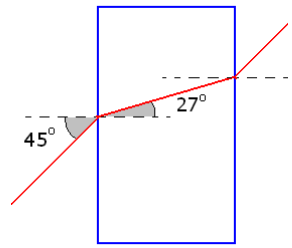
Problem 2.- A triangular prism made of glass (index of refraction n=1.5) is shown in the following figures together with the trajectory of a light ray. Indicate which figure corresponds to the correct trajectory considering refraction.
Justify your answer with a very short rationale.



Solution: The only trajectory that respects Snell's law is the first one. The angle with respect to the normal to the interface must be larger in air than in glass.
In the second and third cases one angle is incorrect and in the fourth case both angles are incorrect.



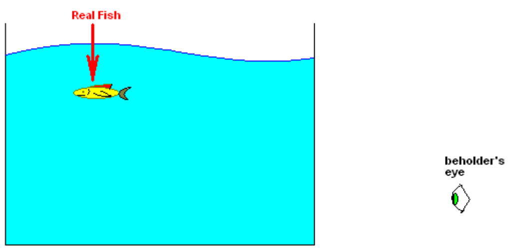
Problem 3.- An experiment with light rays shows the following trajectory through a rectangular glass. Find the index of refraction of the glass.



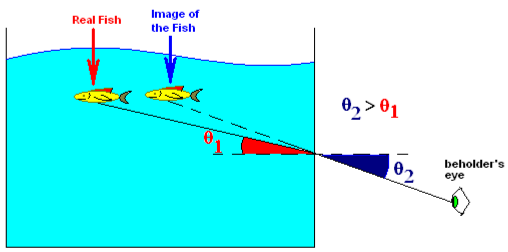
Solution: At the interface we can write Snell's law:

$$n_{air}\sin 45^{\circ} = n_{Glass}\sin 27^{\circ} \rightarrow n_{Glass} = \frac{n_{air}\sin 45^{\circ}}{\sin 27^{\circ}} = \mathbf{1.56}$$

Problem 4.- The figure shows the actual position of a fish in a tank. Indicate the approximate position of the fish **image** as seen by the eye in the following figure:

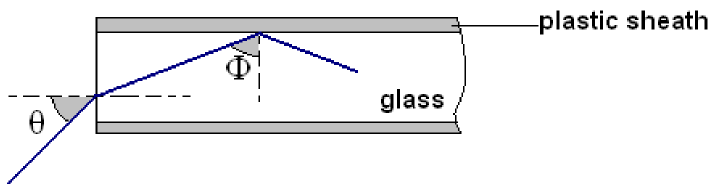


Solution: The angle of incidence of light in water should be smaller than in air because of the index of refraction of water, so the approximate position of the image will be as indicated in the figure below:

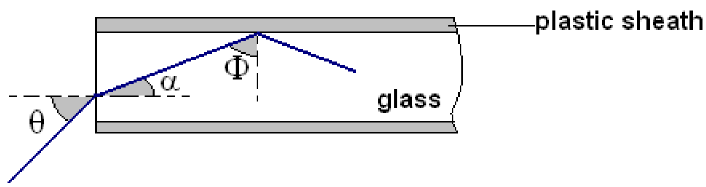


Notice that the real trajectory of light is indicated with a solid line. The person looking at the fish will have the illusion that the fish is at the position of the image.

Problem 5.- For fiber optics to reflect light inside the medium, the angle of incidence on the glass-sheath interface (shown in the figure as Φ) must be greater than 50º. Based on this, calculate the maximum angle θ for incident light that will be reflected internally.
Index of refraction of glass = 1.54



Solution:



According to the problem Φ=50º, so α=40º and we can use Snell's law at the air-glass interface to get:

$$1 \times \sin\theta = 1.54 \times \sin 40^\circ \rightarrow \theta = \sin^{-1}\left(1.54 \sin 40^\circ\right) = \mathbf{77^o}$$

Problem 6.- You point a laser beam to the surface of a swimming pool, making an angle of 45º with respect to the vertical. What angle does the beam make inside the water with respect to the vertical?

Solution: To solve the problem, we use Snell's law $n_i \sin\theta_i = n_r \sin\theta_r$

In this case $\theta_i = 45^\circ$ and the index of refraction of air is approximately 1, while the one of water is 1.33, so:

$$1\sin 45^\circ = 1.33\sin\theta_r \rightarrow \sin\theta_r = \frac{\sin 45^\circ}{1.33} = 0.5317 \rightarrow \theta_r = \sin^{-1}(0.5317) = \mathbf{32.1^o}$$

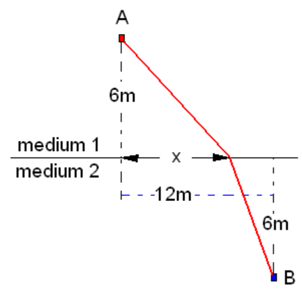
Problem 6a.- A laser beam coming from a submarine exits the water at an angle of 33º to the vertical. What is the angle of incidence of the beam when it hits the air-water interface?
$n_{water} = 1.33$

Solution: This is an application of Snell's Law: $n_{water} \sin\theta_{water} = n_{air} \sin\theta_{air}$

With the values of the problem:

$$1.33\sin\theta_{water} = 1\sin 33^\circ \rightarrow \sin\theta_{water} = \frac{\sin 33^\circ}{1.33} = 0.4095 \rightarrow \theta_{water} = \mathbf{24.2^o}$$

Problem 7.- In the following geometry, find the path that gives the shortest time to get from A to B if the speed in region 1 is v_1=1m/s and the speed in region 2 is v_2=2m/s.



Solution: The geometry of the problem allows us to write down the total time to get from A to B as a function of *x* as follows:

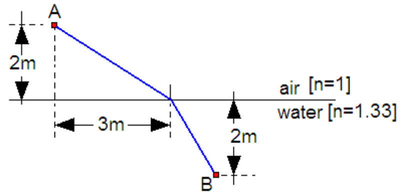
$$Time = \frac{\sqrt{6^2 + x^2}}{1} + \frac{\sqrt{6^2 + (12 - x)^2}}{2}$$

To find the shortest time we take the derivative with respect to *x* and make sure that it is equal to zero:

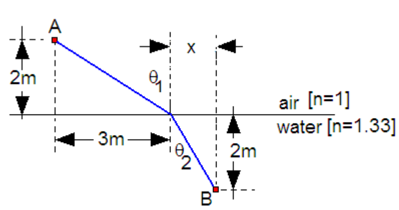
$$\frac{dTime}{dx} = \frac{x}{\sqrt{36 + x^2}} + \frac{-(12 - x)}{2\sqrt{36 + (12 - x)^2}} = 0$$

Solving this equation for *x* gives the best time, which happens when ***x* = 2.77m**

Problem 8.- Determine the time it takes for a beam of light to get from A to B.



Solution: The trajectory of the beam will be as follows:



Snell's Law says that: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Notice that the tangent of angle θ_1 is 3/2, so $\theta_1 = \tan^{-1}(3/2) = 56.31°$

Using Snell's Law: $\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{\sin 56.31^\circ}{1.33} = 0.6256$ so:

$\theta_2 = \sin^{-1}(0.6256) = 38.72^\circ$

The distance traveled in air is $d_{air} = \frac{3m}{\sin\theta_1} = \frac{3}{\sin 56.31^\circ} = 3.61m$

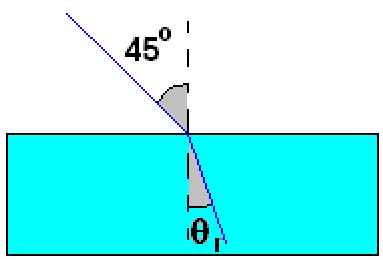
The distance traveled in water is $d_{water} = \frac{2m}{\cos\theta_2} = \frac{2m}{\cos 38.72^\circ} = 2.56m$

The speed of light in air is c
The speed of light in water is c/n=c/1.33

The time to travel from A to B is:

$$\text{time} = \frac{d_{air}}{c} + \frac{d_{water}}{c/1.33} = \frac{3.61m}{3.00\times10^8 m/s} + \frac{2.56m}{2.26\times10^8 m/s} = \mathbf{23.4ns}$$

Problem 9.- You point an ArF excimer laser (wavelength 193 nm in air) to the surface of a UV window with index of refraction 1.48 at an angle of incidence of 45º. Calculate the angle of refraction (θ_r).



Solution: Snell's law:

$$1\sin 45^\circ = 1.48 \sin\theta_r$$

$$\rightarrow \theta_r = \sin^{-1}\left(\frac{\sin 45^\circ}{1.48}\right) = \mathbf{28.5^o}$$