Physics II

Special Relativity

$\beta = \frac{v}{c}$, where $c = 3\times10^8 m/s$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

Length contraction $\mathrm{L} = \frac{\mathrm{L_o}}{\gamma}$, where $\mathrm{L_o}$ is the proper length, measured at rest.

Time dilation $\mathrm{T} = \gamma \mathrm{T_o}$, where $\mathrm{T_o}$ is the proper time, measured at rest.

Problem 1.- A certain unstable particle travels at a speed of $v = 2.4\times10^8 m/s$. At this speed the average lifetime of the particle is 2.7μs. What is the lifetime at rest?

Solution: We find beta: $\beta = \frac{v}{c} = \frac{2.4\times10^8 m/s}{3\times10^8 m/s} = 0.8$

Next, we find gamma: $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.8^2}} = 1.666$

Finally, we find the proper time: $T = \gamma T_o \to T_o = \frac{T}{\gamma} = \frac{2.7\mu s}{1.666} =$ **1.62 μs**

Problem 2.- A particle has a lifetime of 1ns in its own rest frame, but it covers 0.6m in the laboratory before decaying. How fast is it moving?

Solution: We use the basic equation for velocity, but we use the dilated time:

$$v = \frac{d}{t} = \frac{d}{\gamma T_o} = \sqrt{1-\frac{v^2}{c^2}}\frac{d}{T_o}$$

With the numbers of the problem: $v = \frac{0.6}{1\times10^{-9}}\sqrt{1-\frac{v^2}{c^2}} = 6\times10^8\sqrt{1-\frac{v^2}{c^2}}$

To simplify the problem, notice that $6\times10^8 = 2c$, so: $v = 2c\sqrt{1-\frac{v^2}{c^2}}$

It is easier to calculate β writing the equation as: $\beta = 2\sqrt{1-\beta^2}$

Solving for β, we get: $\beta^2 = 4\left(1-\beta^2\right) \to \beta = \sqrt{\frac{4}{5}} = 0.89$

So, **v = 0.89c** or **2.68×10^8 m/s**

Problem 2a.- A particle has a lifetime of 1ns in its own rest frame, but it covers 0.3m in the laboratory before decaying. How fast is it moving?

a) v = c
b) v = 0.81c
c) v = 0.71c
d) v = 0.61c
e) v = 0.51c

Solution: v = 0.71c (**c**)

Problem 3.- If v=0.6c, how much is γ?

a) 1.20
b) 1.25
c) 1.58
d) 1.67

Solution: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.6^2}} = 1.25$

So, if v=0.6c, γ is 1.25 (**b**)

Problem 4.- An object has a length of 6 nm at rest, but it is moving at 60% the speed of light in the direction of its length. How long does it appear?

Solution: Using the value calculated above:

$$L = \frac{L_o}{\gamma} = \frac{6nm}{1.25} = \mathbf{4.8\ nm}$$

Problem 4a.- An object that has a length of 25 cm at rest, but it is moving at 28% the speed of light. How long does it appear?

a) 24 cm
b) 23 cm
c) 22 cm
d) 21 cm
e) 20 inches

Solution: $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.28^2}} = 1.04$

$$L = \frac{25cm}{1.04} = 24cm \ (\mathbf{a})$$

Problem 5.- Neutrons have a half-life at rest of 608 s. What would be their half-life in motion with v = 0.8c?

a) 365 s
b) 608 s
c) 1013 s
d) 1216 s

Solution: If they move at v = 0.8c their lifetime is 1013 s (**c**)

Problem 6.- A nanowire of length 3nm is accelerated to a high velocity in the direction of its length. It is so fast that it seems to be only 1.8nm long. How fast is it moving?

a) v = 0.6c
b) v = 0.7c
c) v = 0.8c
d) v = 0.9c

Solution: Since L=1.8nm and L_o=3nm the value of gamma is 3/1.8=1.66, so:

$$\gamma = 1.66 = \frac{1}{\sqrt{1-\beta^2}} \rightarrow 1.66^2 = \frac{1}{1-\beta^2} \rightarrow 1.66^2 - 1.66^2\beta^2 = 1 \rightarrow \beta^2 = \frac{1.66^2 - 1}{1.66^2}$$

$$\beta = \sqrt{\frac{1.66^2 - 1}{1.66^2}} = 0.8\text{c} \quad (\mathbf{c})$$

Problem 7.- At high speeds it becomes more difficult to accelerate an object. What is the best explanation of this phenomenon?

a) The number of atoms in the object increases.
b) The object acquires more mass at high speeds.
c) Linear momentum increases beyond the classical value p=mv
d) $E=ma^2$
e) $E=mb^2$
f) $E=mc^2$

Solution: Linear momentum increases beyond the classical value p = mv (**c**)

Problem 8.- How much energy is contained in 1kg of mass at rest?

a) 9×10^{16} joule
b) 4.5×10^{16} joule
c) 0 joule
d) 9×10^{16} eV
e) 4.5×10^{16} eV

Solution: $E=mc^2$, so in 1kg of mass $E=9\times10^{16}$ joule (**a**)