Quantum Mechanics

Kinetic energy

Problem 1.- Find the expectation value of the kinetic energy of a particle with this wave function:

$$\psi(x)=\begin{cases}\sqrt{\frac{3}{2a}}\sin\left(\frac{\pi x}{a}\right)+\sqrt{\frac{1}{2a}}\sin\left(\frac{5\pi x}{a}\right) & if \quad 0<x<a\\ 0 & otherwise\end{cases}$$

Solution: The kinetic energy operator is $\frac{p^2}{2m}=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

So the expectation value of the kinetic energy is given by:

$$\langle K.E.\rangle=\int_0^a\psi^*\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi dx$$

Given the function of the problem:

$$\langle K.E.\rangle=\int_0^a\left(\sqrt{\frac{3}{2a}}\sin\left(\frac{\pi x}{a}\right)+\sqrt{\frac{1}{2a}}\sin\left(\frac{5\pi x}{a}\right)\right)^*\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\left(\sqrt{\frac{3}{2a}}\sin\left(\frac{\pi x}{a}\right)+\sqrt{\frac{1}{2a}}\sin\left(\frac{5\pi x}{a}\right)\right)dx$$

Taking the derivative:

$$\langle K.E.\rangle=\int_0^a\left(\sqrt{\frac{3}{2a}}\sin\left(\frac{\pi x}{a}\right)+\sqrt{\frac{1}{2a}}\sin\left(\frac{5\pi x}{a}\right)\right)\left(\sqrt{\frac{3}{2a}}\frac{\hbar^2\pi^2}{2ma^2}\sin\left(\frac{\pi x}{a}\right)+\sqrt{\frac{1}{2a}}\frac{25\hbar^2\pi^2}{2ma^2}\sin\left(\frac{5\pi x}{a}\right)\right)dx$$

After multiplication we find four integrands, but the cross products give zero upon integration. The other two terms give:

$$\langle K.E.\rangle=\int_0^a\frac{3}{2a}\frac{\hbar^2\pi^2}{2ma^2}\sin^2\left(\frac{\pi x}{a}\right)+\frac{1}{2a}\frac{25\hbar^2\pi^2}{2ma^2}\sin^2\left(\frac{5\pi x}{a}\right)dx$$

Computing the integrals:

$$\langle K.E.\rangle=\frac{a}{2}\frac{3}{2a}\frac{\hbar^2\pi^2}{2ma^2}+\frac{a}{2}\frac{1}{2a}\frac{25\hbar^2\pi^2}{2ma^2}\rightarrow\langle K.E.\rangle=\frac{7\hbar^2\pi^2}{2ma^2}$$